Solucionario

Trigonometría

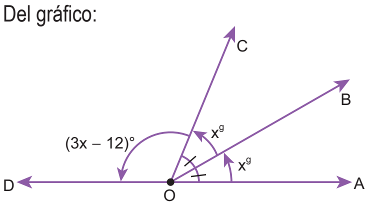
3.°

Unidad 1

SISTEMAS DE MEDICIÓN ANGULAR

APLICAMOS LO APRENDIDO (página 6) Unidad 1

1. Del gráfico:



Luego:

$$(3x-12)° + (2x)^g = 180°$$

$$(3x-12)° + (\cancel{2}x)^g \cdot \frac{9°}{\cancel{10}^g} = 180°$$

$$(3(x-4))° + \left(\frac{9x}{5}\right)° = 180°$$

$$(x-4) + \left(\frac{3x}{5}\right) = 60$$

$$5x - 20 + 3x = 300$$
$$8x = 320$$
$$x = 40$$

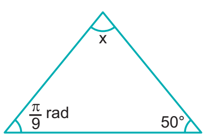
Luego:

$$(2x)^g = (2\times 40)^g = 80^g = 80^g \times \frac{9°}{10^g} = 72°$$

$\therefore\ (2x)^g = 72°$

Clave D

2.



$$\frac{\pi}{9}\text{ rad}\cdot\left(\frac{180°}{\pi\text{ rad}}\right) = \frac{180°}{9} = 20°$$

Por propiedad:

$$x + \frac{\pi}{9}\text{ rad} + 50° = 180°$$
$$x + 20° + 50° = 180°$$
$$\therefore\ x = 110°$$

Clave B

3. Sea el ángulo: $\alpha = \frac{\pi}{3}$ rad

Su suplemento será: $180° - \alpha$

Por dato:

$$180° - \alpha = 2x + 10°$$

$$180° - \frac{\pi}{3}\text{ rad} = 2x + 10°$$

$$180° - \frac{\pi}{3}\text{ rad}\left(\frac{180°}{\pi\text{ rad}}\right) = 2x + 10°$$

$$180° - 60° = 2x + 10°$$
$$110° = 2x$$
$$\therefore\ x = 55°$$

Clave E

4. $\beta = \frac{\pi}{6}\text{ rad} - 30^g$

$$\beta = \frac{\pi}{6}\text{ rad}\frac{(180°)}{(\pi\text{ rad})} - 30^g\frac{(9°)}{(10^g)}$$

$$\beta = \frac{180°}{6} - \frac{30\times 9°}{10}$$

$$\beta = 30° - 27° = 3°$$

$\therefore\ \beta = 3°$

Clave A

5. Sea el ángulo: α

Por dato:

$\alpha = (x-1)° \ \wedge\ \alpha = (x+1)^g$

Igualando:

$$(x-1)° = (x+1)^g$$

$$(x-1)°\left(\frac{10^g}{9°}\right) = (x+1)^g$$

$$10x - 10 = 9x + 9$$
$$x = 19$$

Entonces:

$$\alpha = (x-1)° = (19-1)°$$

$$\alpha = 18°\cdot\left(\frac{\pi\text{ rad}}{180°}\right) = \frac{\pi}{10}\text{ rad}$$

$$\therefore\ \alpha = \frac{\pi}{10}\text{ rad}$$

Clave B

6. $x°y'z'' = 3°36'34'' + 2°28'42''$

$x°y'z'' = 5°64'76''$

$x°y'z'' = 5°65'16''$

$x°y'z'' = 6°5'16''$

$\Rightarrow x = 6;\ y = 5;\ z = 16$

Piden: $x + y + z = 6 + 5 + 16 = 27$

$\therefore\ x + y + z = 27$

Clave A

7. $E = \dfrac{6\pi C - 5\pi S + 20R}{\pi C - 40R}$

Se cumple:

$$\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi} = k$$

$\Rightarrow S = 180k;\ C = 200k;\ R = \pi k$

Reemplazando:

$$E = \frac{6\pi(200k) - 5\pi(180k) + 20(\pi k)}{\pi(200k) - 40(\pi k)}$$

$$E = \frac{1200 - 900 + 20}{200 - 40} = \frac{320}{160} = 2$$

$\therefore\ E = 2$

Clave E

8. Sean los ángulos: α y β

Del enunciado:

$\alpha + \beta = 60^g = 54°$

$\alpha - \beta = \frac{\pi}{10}\text{rad} = 18°$

Entonces:

$$\left.\begin{aligned}\alpha + \beta &= 54°\\ \alpha - \beta &= 18°\end{aligned}\right\}(+)$$

$$2\alpha = 72°$$
$$\alpha = 36°$$
$$\Rightarrow \beta = 18°$$

Por lo tanto, el ángulo mayor mide: 36°

Clave A

9. $E = \dfrac{1°}{1'} - \dfrac{1^g}{1^m} + \dfrac{1'}{1''}\cdot\dfrac{1^m}{1^s}$

$$E = \frac{(60')}{1'} - \frac{(100^m)}{1^m} + \frac{(60'')}{1''}\cdot\frac{(100^s)}{1^s}$$

$E = 60 - 100 + 60 \cdot 100$

$E = 60 - 100 + 6000 = 5960$

$\therefore\ E = 5960$

Clave E

10. $\dfrac{10}{9C} - \dfrac{9}{10S} = \dfrac{R}{2\pi}$ (dato) ...(1)

Se cumple:

$$\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi}$$

$$\Rightarrow C = \frac{10S}{9};\ R = \frac{\pi S}{180}$$

Reemplazando en (1):

$$\frac{10}{9\left(\frac{10S}{9}\right)} - \frac{9}{10S} = \frac{\frac{\pi S}{180}}{2\pi}$$

$$\frac{10}{10S} - \frac{9}{10S} = \frac{\pi S}{360\pi}$$

$$\frac{1}{10S} = \frac{S}{360}$$

$S^2 = 36 \ \Rightarrow S = 6$

$\therefore$ La medida sexagesimal del ángulo es 6°.

Clave A

11. Sean los ángulos: α, β y θ

Del enunciado:

$\underbrace{\alpha;\ \beta}_{+20°};\ \underbrace{\ \theta}_{+20°} \ \wedge \quad \beta + \theta = 200°$

$+20° + 20°$

$$\beta + (\beta + 20°) = 200°$$
$$2\beta = 180°$$
$$\beta = 90°$$

Luego:

$\beta = 90° \Rightarrow \alpha = 70° \ \wedge\ \theta = 110°$

Entonces:

$$\alpha + \beta + \theta = 70° + 90° + 110° = 270°\cdot\left(\frac{10^g}{9°}\right)$$

$\therefore\ \alpha + \beta + \theta = 300^g$

Clave C

12. $\alpha = 17^g\cdot\left(\frac{9°}{10^g}\right) = 15{,}3°$

$\beta = 180°$

$\theta = \frac{\pi}{12}\text{ rad}\left(\frac{180°}{\pi\text{ rad}}\right) = 15°$

Los ángulos en forma creciente serán: $\theta;\ \alpha;\ \beta$.

Clave B

13. $E = \sqrt{\dfrac{5S - 4C}{C - S} + \sqrt{\dfrac{C + S}{C - S} - 3}}$ ...(1)

Se cumple:

$$\frac{S}{9} = \frac{C}{10} \Rightarrow S = 9k \ \wedge\ C = 10k$$

Reemplazando en (1):

$$E = \sqrt{\frac{5(9k) - 4(10k)}{(10k) - (9k)} + \sqrt{\frac{(10k) + (9k)}{(10k) - (9k)} - 3}}$$

$E = \sqrt{5 + \sqrt{19 - 3}} = \sqrt{5 + \sqrt{16}}$

$E = \sqrt{5 + 4} = \sqrt{9} = 3 \quad \therefore\ E = 3$

Clave B

14. Del triángulo isósceles: $m\angle A = m\angle C = \alpha$, luego:

$$2\alpha + 108° = 180°$$
$$2\alpha = 72°$$
$$\alpha = 36°$$

Luego:

$S = 36$; además C es el número de grados centesimales del ángulo α, entonces se cumple:

$$\frac{C}{10} = \frac{S}{9} \Rightarrow C = \frac{10}{9}(36)$$

$$C = 40$$

Reemplazando en M:

$M = 36^g + 40°;$

$M = 36^g \times \frac{\pi \text{ rad}}{200^g} + 40° \times \frac{\pi}{180°} \text{ rad}$

$M = \left(\frac{9\pi}{50} + \frac{2\pi}{9}\right) \text{rad}$

$\therefore\ M = \frac{181\pi}{450} \text{ rad}$

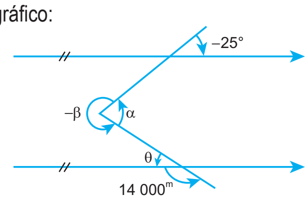
Clave C

PRACTIQUEMOS

Nivel 1 (página 8) Unidad 1

Comunicación matemática

1. Del gráfico:



$180° = 14\,000^m + \theta$

$180° = 140 \times 100^m + \theta$

$180° = 140^g + \theta$

$180° = 140^g \times \frac{9°}{10^g} + \theta$

$180° = 126° + \theta \Rightarrow \theta = 54°$

Por propiedad

$\alpha = 25° + \theta = 25° + 54°$

$\alpha = 79° \quad ... (1)$

Además:

$-\beta + \alpha = 360° \Rightarrow -\beta + 79° = 360°$

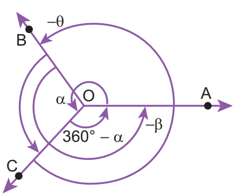
$\beta = 79° - 360°$

$\therefore\ \beta = -281° \quad ... (2)$

- De (1) y (2): $\beta < \alpha;\ -281° < 79°$; I (F)
- De (1): $\alpha = 79°$; II (F)
- De (2): $\beta = -281°$; III (F)

Clave B

2. En el gráfico:



El $\angle COA = 360° - \alpha$, entonces:

$\angle BOC = -\beta - (360° - \alpha)$

$\angle BOC = -\beta - 360° + \alpha \quad ... (1)$

Finalmente:

$\angle BOC + \angle COB = 360°$

De (1):

$-\beta - 360° + \alpha + (-\theta) = 360°$

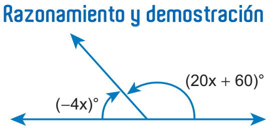
$-\beta + \alpha - \theta = 2 \times 360°$

$\therefore\ \alpha - \beta - \theta = 720°$

Clave A

Razonamiento y demostración

3.



$(20x + 60)° - (-4x)° = 180°$

$20x + 60 + 4x = 180$

$24x = 120$

$\therefore x = 5$

Clave E

4. Del gráfico:

$\alpha - \beta + \theta = 180°$

Clave E

5. I. $450^g \times \frac{9°}{10^g} = 405°$

Clave C

II. $\frac{\pi}{6} \text{ rad} \times \frac{180°}{\pi \text{ rad}} = 30°$

Clave B

6. $E = \frac{2°9'}{3'} + \frac{1^g25^m}{5^m}$

$E = \frac{120' + 9'}{3'} + \frac{100^m + 25^m}{5^m}$

$E = \frac{129}{3} + \frac{125}{5}$

$E = 43 + 25 = 68$

Clave C

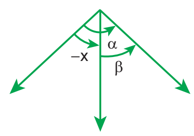
7. I. $270° \times \frac{10^g}{9°} = 300^g$

Clave A

II. $\frac{\pi}{10} \text{ rad} \times \frac{200^g}{\pi \text{ rad}} = 20^g$

Clave D

8.



$\alpha = \beta - x$

$\therefore x = \beta - \alpha$

Clave D

9. $P = 40^g + \frac{3\pi}{4} \text{ rad}$

Todo al sistema centesimal:

$\frac{C}{10} = \frac{20R}{\pi} \Rightarrow \frac{C}{10} = \frac{20}{\pi} . \frac{3\pi}{4} \Rightarrow C = 150^g$

$P = 40^g + 150^g = 190^g$

Al sistema sexagesimal:

$\frac{S}{9} = \frac{C}{10} \Rightarrow \frac{S}{9} = \frac{190}{10}$

$\Rightarrow S = 171$

$\therefore\ P = 171°$

Clave A

10. $J = \frac{3°5'}{5'}$

$1° <> 60'$

$\Rightarrow \quad 3° <> 180'$

$J = \frac{180' + 5'}{5'} \qquad \therefore\ J = \frac{185'}{5'} = 37$

Clave C

11. $E = \dfrac{30°}{\frac{\pi}{12} \text{ rad}} + \dfrac{40^g}{\frac{\pi}{5} \text{ rad}}$

30° lo pasamos a radianes:

$\frac{S}{9} = \frac{20R}{\pi} \Rightarrow \frac{30}{9} = \frac{20R}{\pi}$

$R = \frac{\pi}{6} \text{ rad}$

40^g lo pasamos a radianes:

$\frac{C}{10} = \frac{20R}{\pi} \Rightarrow \frac{40}{10} = \frac{20R}{\pi}$

$R = \frac{4\pi}{20} = \frac{\pi}{5} \text{ rad}$

$E = \dfrac{\frac{\pi}{6} \text{ rad}}{\frac{\pi}{12} \text{ rad}} + \dfrac{\frac{\pi}{5} \text{ rad}}{\frac{\pi}{5} \text{ rad}}$

$E = \frac{12}{6} + \frac{5}{5} = 2 + 1 = 3$

Clave C

Resolución de problemas

12. $\frac{\pi}{9} \text{ rad} + \frac{\pi}{3} \text{ rad} + x = 180°$

$\frac{4\pi}{9} \text{ rad} + x = 180°$

$\frac{4\pi}{9} \text{ rad} \times \frac{180°}{\pi \text{ rad}} + x = 180°$

$80° + x = 180°$

$\therefore x = 100°$

Clave C

13. $(80n)^g + (18n)° + \frac{\pi n}{3} \text{ rad} = 180°$

$(80n)^g \times \frac{9°}{10^g} + (18n)° + \frac{\pi n}{3} \text{ rad} \times \frac{180°}{\pi \text{ rad}} = 180°$

$72n + 18n + 60n = 180$

$150n = 180$

$n = \frac{6}{5}$

Clave C

14. $(7n - 4)° = (8n - 6)^g$

$(7n - 4)° \times \frac{10^g}{9°} = (8n - 6)^g$

$(7n - 4) \times 10^g = 9(8n - 6)^g$

$70n - 40 = 72n - 54$

$-2n = -14$

$\therefore n = 7$

Clave D

15. $\frac{\pi}{3} \text{ rad} + 40^g + x = 180°$

$\frac{\pi}{3} \text{ rad} \times \frac{180°}{\pi \text{ rad}} + 40^g \times \frac{9°}{10^g} + x = 180°$

$60° + 36° + x = 180°$

$\therefore x = 84°$

Clave C

16. Por dato:

$S = 45°$

$C = 50^g$

Además:

$S = 9k$ y $C = 10k$

Reemplazando:

$45 = 9k$ y $50 = 10k$

Nos piden:

$\frac{S + 35}{C + 14} = \frac{45 + 35}{50 + 14} = \frac{80}{64} = \frac{5}{4}$

Clave D

17. Un ángulo mide $(7n + 3)°$ y también $(8n + 2)^g$:

$\frac{7n + 3}{9} = \frac{8n + 2}{10}$

$70n + 30 = 72n + 18$

$12 = 2n$

$n = 6$

$S = 7(6) + 3$

$S = 45°$

$\Rightarrow \frac{S}{9} = \frac{20R}{\pi}$

$\frac{45}{9} = \frac{20R}{\pi}$

$\frac{5\pi}{20} = R \Rightarrow R = \frac{\pi}{4} \text{ rad}$

Clave B

18. Sean x, y dos ángulos complementarios.

$x + y = 90°$

Por dato: $x - y = 49^g$

49^g a sexagesimal:

$\frac{S}{9} = \frac{C}{10} \Rightarrow \frac{S}{9} = \frac{49}{10}$

$S = \frac{441°}{10}$

$\left.\begin{aligned} x + y &= 90° \\ x - y &= \frac{441°}{10} \end{aligned}\right\}(-)$

$2y = 90° - \frac{441°}{10} = \frac{900° - 441°}{10}$

$2y = \frac{459°}{10} \Rightarrow y = \frac{459°}{20} \times \frac{\pi}{180°}$

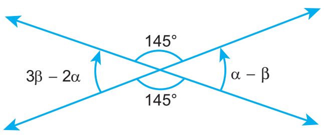
$\therefore y = \frac{51\pi}{400}$ rad

Clave B

Nivel 2 (página 9) Unidad 1

Comunicación matemática

19.



Del gráfico:

$-3\beta + 2\alpha + 290° + \alpha - \beta = 360°$

$-4\beta + 3\alpha = 70°$ ...(1)

Además:

$\alpha - \beta = -3\beta + 2\alpha$

$2\beta = \alpha$ ...(2)

Reemplazando (2) en (1):

$-4\beta + 3(2\beta) = 70°$

$2\beta = 70° \Rightarrow \beta = 35°$

$\therefore \alpha = 70°$

Entonces:

$3\alpha + \frac{30}{7}\beta = 3(70°) + \frac{30}{7}(35°)$

$= 210° + 150°$

$= 360°$ (1 vuelta)

Clave A

20. $A = \frac{\pi}{4}$ rad + 10°

$\frac{\pi}{4}$ rad al sistema sexagesimal:

$\frac{S}{9} = \frac{20R}{\pi} \Rightarrow \frac{S}{9} = \frac{20}{\pi}\left(\frac{\pi}{4}\right) \Rightarrow S = 45$

$A = 45° + 10° = 55°$

$B = \frac{\pi}{5}$ rad + 30°

$\frac{\pi}{5}$ rad al sistema sexagesimal:

$\frac{S}{9} = \frac{20R}{\pi} \Rightarrow \frac{S}{9} = \frac{20}{\pi}\left(\frac{\pi}{5}\right) \Rightarrow S = 36$

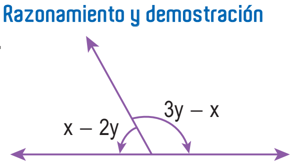
$B = 30° + 36° = 66°$

Por lo tanto, A es menor que B.

Clave B

Razonamiento y demostración

21.



Suman 180° porque forman un ángulo llano:

$x - 2y - (3y - x) = 180°$

$x - 2y - 3y + x = 180°$

$2x - 5y = 180°$

$\therefore \left(\frac{2x - 5y}{4}\right) = 45°$

Clave A

22. $\alpha = \frac{7\pi}{12}$ rad + 36°

Todo al sistema sexagesimal:

$\frac{S}{9} = \frac{20R}{\pi}$

$\frac{S}{9} = \frac{20}{\pi}\left(\frac{7\pi}{12}\right) \Rightarrow S = \frac{20.7.9}{12} = 105$

$\alpha = 105° + 36° = 141°$

Clave C

23. $E = \frac{1^g}{10^m} + \frac{1°}{3'} + \frac{1^m}{1^s}$

$1^g <> 100^m \quad 1° <> 60' \quad 1^m <> 100^s$

$E = \frac{100^m}{10^m} + \frac{60'}{3'} + \frac{100^s}{1^s}$

$E = 10 + 20 + 100 = 130$

Clave E

Resolución de problemas

24. $\frac{3\pi}{11}$ rad $= \overline{4a}°\, b'\, \overline{2c}''$

$1° <> 60' \quad 1' <> 60''$

$\frac{3\pi}{11}$ rad al sistema sexagesimal:

$\frac{S}{9} = \frac{20R}{\pi} \Rightarrow \frac{S}{9} = \frac{20}{\pi}\left(\frac{3\pi}{11}\right) \Rightarrow S = \frac{540}{11}$

540° | 11
44°   49° 5' 27"
100°
99°
1° <> 60'
55
5' <> 300"
22"
80"
77"
3"

$\Rightarrow a = 9 \quad b = 5 \quad c = 7$

$L = \frac{ab}{c - 2} = \frac{9 \times 5}{7 - 2} = \frac{9 \times 5}{5} = 9$

Clave E

25. Sean x, y ángulos complementarios:

$\Rightarrow \left.\begin{aligned} x + y &= 90° \\ x - y &= 18° \end{aligned}\right\}(+)$

$2x = 108°$

$x = 54°$

$\Rightarrow 54° + y = 90°$

$y = 36°$

$\therefore$ El menor es: 36°

Clave A

26. Dos ángulos son: 18°; $0{,}25\pi$ rad, a sexagesimal:

$\frac{S}{9} = \frac{20R}{\pi} \Rightarrow \frac{S}{9} = \frac{20}{\pi} \cdot \frac{25\pi}{100} \Rightarrow S = 45$

$18° + 45° = 63°$

En un triángulo la medida de los tres ángulos suman 180°.

$\therefore$ El tercer ángulo es: $180° - 63° = 117°$

Clave C

27. $\frac{S}{90} + \frac{C}{50} + \frac{R}{\pi} = 14$

Recuerda:

$S = 9k$; $C = 10k$; $R = \frac{\pi}{20}k$

$\frac{9k}{90} + \frac{10k}{50} + \frac{\frac{\pi k}{20}}{\pi} = 14$

$\frac{k}{10} + \frac{k}{5} + \frac{k}{20} = 14$

$\frac{2k + 4k + k}{20} = 14 \Rightarrow \frac{7k}{20} = 14$

$k = 40$

$\Rightarrow R = \frac{\pi}{20}(40) = 2\pi$

$\therefore$ El ángulo es 2π rad.

Clave B

28. Un ángulo en sexagesimal: $S = 9k$

Un ángulo en centesimal: $C = 10k$

$9k + 10k = 133$

$19k = 133 \Rightarrow k = 7$

$S = 9(7) = 63$; $C = 10(7) = 70$

Por lo tanto, el ángulo es 63° ó 70^g.

Clave E

29. Un ángulo en sexagesimal: $\left(\frac{25}{x} + 2\right)^{o}$

El mismo ángulo en centesimal: 280^g

280^g a sexagesimal:

$\frac{S}{9} = \frac{C}{10} \Rightarrow \frac{S}{9} = \frac{280}{10}$

$\Rightarrow S = 252$

Luego:

$\left(2 + \frac{25}{x}\right)^{o} = 252°$

$\frac{25}{x} = 250$

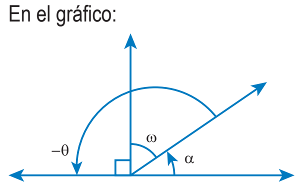
$x = \frac{1}{10} \Rightarrow x = 0{,}1$

Clave B

Nivel 3 (página 10) Unidad 1

Comunicación matemática

30. En el gráfico:



Se observa:

$\alpha + \omega = 90°$

$\alpha = 90° - \omega \Rightarrow \alpha < 90°$

Además:

α gira en sentido antihorario $\Rightarrow \alpha > 0°$

$\therefore 0° < \alpha < 90°$ ... (1)

El ángulo α es menor que 90°. I. (V)

Luego:

$\alpha - \theta = 180°$

$\alpha = 180° + \theta$

De (1):

$0° < \alpha < 90°$
$0° < 180° + \theta < 90°$
$-180° < \theta < -90°$

El ángulo $\theta \in \langle -180°, -90° \rangle$; II. (V)

Análogamente:
$\sup(\alpha) = 180° - \alpha$
$\alpha = 180° - \sup(\alpha)$

De (1):

$0° < \alpha < 90°$
$0° < 180° - \sup(\alpha) < 90°$
$-180° < -\sup(\alpha) < -90°$
$90° < \sup(\alpha) < 180°$

El suplemento de α $(\sup(\alpha)) \in \langle 90°; 180° \rangle$ III. (V)

Clave D

31. Del enunciado:

a: minutos sexagesimales $\Rightarrow a = 60S$
b: minutos centesimales $\Rightarrow b = 100C$
c: segundos sexagesimales $\Rightarrow c = 3600S$
d: segundos centesimales $\Rightarrow d = 10\,000C$

Donde C y S son los números de grados en el sistema centesimal y sexagesimal, respectivamente.

Luego:

- $\frac{c}{a} = \frac{3600\,S}{60\,S} = 60$

  $\therefore \frac{c}{a} = 60$

- $\frac{d}{b} = \frac{10\,000\,C}{100\,C} = 100$

  $\therefore \frac{d}{b} = 100$

- $\frac{a}{b} = \frac{60\,S}{100\,C} = \frac{3}{5} \cdot \frac{S}{C}$;

  Sabemos: $\frac{S}{C} = \frac{9}{10}$

  $\Rightarrow \frac{a}{b} = \frac{3}{5} \cdot \frac{9}{10}$

  $\therefore \frac{a}{b} = \frac{27}{50}$

- $\frac{d}{c} = \frac{10\,000\,C}{3600\,S} = \frac{25}{9} \cdot \frac{C}{S}$

  $\Rightarrow \frac{d}{c} = \frac{25}{9} \cdot \frac{10}{9}$

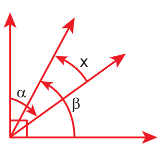
  $\therefore \frac{d}{c} = \frac{250}{81}$

Relacionando se tiene que:
Ib; IIa; IIId; IVc

Clave B

Razonamiento y demostración

32.



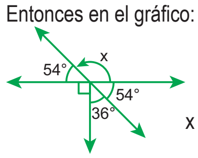
$\beta - x - \alpha = 90°$
$-x = 90° + \alpha - \beta$
$\therefore x = \beta - \alpha - 90°$

Clave A

33. Convirtiendo 40^g a grados sexagesimales:

$$40^g \times \frac{9°}{10^g} = 36°$$

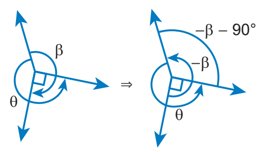
Entonces en el gráfico:



$x + 54° + 90° + 90° = 360°$
$x = 126°$

Clave B

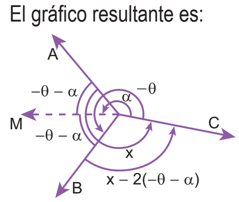
34.



Entonces del segundo gráfico:
$\theta - \beta - 90° = 360°$
$\therefore \theta - \beta = 450°$

Clave C

35. El gráfico resultante es:



Del gráfico resultante:
$\alpha - \theta - \alpha + x - 2(-\theta - \alpha) = 360°$
$\alpha - \theta - \alpha + x + 2\theta + 2\alpha = 360°$
$2\alpha + \theta + x = 360°$
$\therefore x = 360° - 2\alpha - \theta$

Clave A

36.
$$\frac{\frac{S^5}{81}+\frac{C^4}{100}+400\frac{R^3}{\pi^2}}{\frac{S^4}{36}+\frac{C^3}{40}+5\frac{R^2}{\pi}} = \frac{S}{3}+\frac{C}{4}-5$$

$$\frac{S}{9} = \frac{C}{10} = \frac{20R}{\pi} = k$$

$S = 9k$; $C = 10k$; $R = \frac{\pi k}{20}$

$$\frac{\frac{(9k)^5}{9^2}+\frac{(10k)^4}{10^2}+\frac{400}{\pi^2}\frac{(\pi k)^3}{20^3}}{\frac{(9k)^4}{36}+\frac{(10k)^3}{40}+5\frac{\left(\frac{\pi k}{20}\right)^2}{\pi}} = \frac{9k}{3}+\frac{10k}{4}-5$$

$$\frac{9^3k^5+10^2k^4+\frac{\pi k^3}{20}}{\frac{9^4.k^4}{9.4}+\frac{10^3k^3}{40}+5\frac{\pi.k^2}{400}} = 3k+\frac{5}{2}k-5$$

$$\frac{k^3\left(9^3k^2+10^2k+\frac{\pi}{20}\right)}{\frac{9^3k^4}{4}+\frac{10^2k^3}{4}+\frac{\left(\frac{\pi}{20}\right)}{4}k^2} = \frac{11k}{2}-5$$

$$\frac{k^3\left(9^3k^2+10^2k+\frac{\pi}{20}\right)}{\frac{k^2\left(9^3k^2+10^2k+\frac{\pi}{20}\right)}{4}} = \frac{11k}{2}-5$$

$4k = \frac{11k}{2} - 5 \Rightarrow k = \frac{10}{3}$

$R = \frac{\pi}{20}k = \frac{\pi}{20}\left(\frac{10}{3}\right) = \frac{\pi}{6}$

Por lo tanto, la medida circular es $\frac{\pi}{6}$ rad

Clave D

Resolución de problemas

37. Se tiene de la fórmula de conversión:

$\frac{S}{C} = \frac{C}{10}$

Reemplazando:

$S = \frac{9}{10} \cdot (19{,}375)$

$S = 17{,}4375$

Luego:
$(17{,}4375)° = 17° + (0{,}4375) \times 1°$
$= 17° + (0{,}4375) \times 60'$
$= 17° + (26{,}25)'$
$= 17° + 26' + 0{,}25'$
$= 17° + 26' + 0{,}25 \times 60''$
$= 17° + 26' + 15''$

Entonces: $19{,}375^g = 17°\ 26'\ 15''$
$a = 17$; $b = 26$; $c = 15$
$a + b + c = 17 + 26 + 15 = 58$

Finalmente:
$(a + b + c)° = 58° = 58° . \frac{\pi \text{ rad}}{180°}$

$\therefore (a + b + c)° = \frac{29\pi}{90}$ rad

Clave A

38. Sean M y n los ángulos mayor y menor, respectivamente:

$(90° - M) + (90° - n) = 70^g$
$180° - (M + n) = 70^g \times \frac{9°}{10^g}$
$180° - (M + n) = 63°$
$M + n = 117°$ ... (1)

Además:
$M - n = 13°$ ... (2)
De (1) y (2):
$M - n = 13°$
$M + n = 117°$ (+)
$2M = 130°$
$M = 65°$

Reemplazando en (2):
$65° - n = 13°$
$n = 52°$

Se pide:
$\sqrt{2n + M} = \sqrt{2(52°) + 65°}$
$= \sqrt{169}$
$\therefore \sqrt{2n + M} = 13°$

Clave E

39. $\theta = 26°\ 12'\ 45'' = 26° + 12' + 45'' . \frac{1'}{60''}$

$\theta = 26° + 12' + 0{,}75' = 26° + 12{,}75' . \frac{1°}{60'}$

$\theta = 26° + 0{,}2125° = 26{,}2125°$

$\theta = 26{,}2125° . \frac{10^g}{9°} = 29{,}125^g$

$\theta = 29^g + 0{,}125^g . \frac{100^m}{1^g} = 29^g + 12{,}5^m$

$\theta = 29^g + 12^m + 0{,}5^m . \frac{100^s}{1^m} = 29^g + 12^m + 50^s$

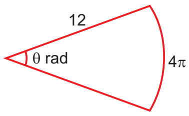
$\therefore 26°\ 12'\ 45'' <> 29^g\ 12^m\ 50^s$

Clave E

SECTOR CIRCULAR

APLICAMOS LO APRENDIDO
(página 11) Unidad 1

1.



Se cumple:

$L = \theta \cdot r$

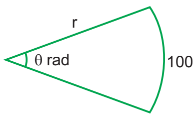
$4\pi = \theta \cdot 12$

$\theta = \dfrac{4\pi}{12}$

$\therefore\ \theta = \dfrac{\pi}{3}$ rad

Clave B

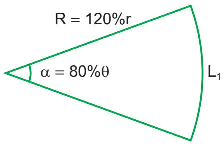
2.



Se cumple:

$L = \theta \cdot r$

$100 = \theta \cdot r$ ...(I)



Se cumple:

$L_1 = \alpha \cdot R$

$L_1 = (80\%\theta)(120\%r)$

$L_1 = 0{,}96(\theta \cdot r)$ ...(II)

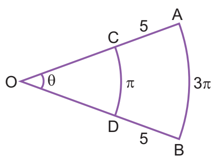
Reemplazando (I) en (II):

$L_1 = 0{,}96(100) = 96$

$\therefore\ L_1 = 96$ m

Clave A

3.



Por propiedad, en el trapecio circular:

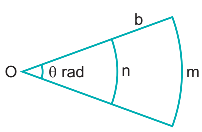
$\theta = \dfrac{3\pi - \pi}{5} = \dfrac{2\pi}{5}$ rad

$\theta = \dfrac{2\pi}{5}\text{ rad}\cdot\left(\dfrac{180°}{\pi\text{ rad}}\right) = \left(\dfrac{180° \times 2}{5}\right) = 72°$

$\therefore\ \theta = 72°$

Clave C

4.



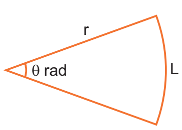
Por propiedad, en el trapecio circular:

$\theta = \dfrac{m-n}{b} \Rightarrow 1 = \dfrac{m-n}{\theta \cdot b}$

$\therefore\ \dfrac{m-n}{b\theta} = 1$

Clave B

5.



Se cumple:

$L = \theta \cdot r$ ...(I)

Por dato:

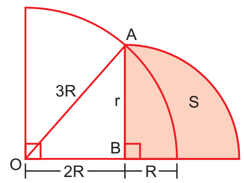
$\dfrac{r}{L} = \dfrac{2}{3} \Rightarrow L = \dfrac{3}{2}r$

Reemplazando en (I):

$\dfrac{3}{2}r = \theta \cdot r \Rightarrow \theta = \dfrac{3}{2}$ rad

Clave D

6.



En el ⊿OBA, por el teorema de Pitágoras:

$(3R)^2 = (2R)^2 + r^2 \Rightarrow r^2 = 5R^2$

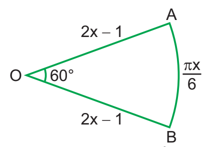
Luego:

$S = \dfrac{\left(\frac{\pi}{2}\right)r^2}{2} = \dfrac{\pi r^2}{4} = \dfrac{\pi(5R^2)}{4}$

$\therefore\ S = \dfrac{5\pi}{4}R^2$

Clave C

7.



$60° = 60°\cdot\left(\dfrac{\pi\text{ rad}}{180°}\right) = \dfrac{\pi}{3}$ rad

Se cumple:

$L = \theta \cdot R \Rightarrow \left(\dfrac{\pi}{3}\right)(2x-1) = \dfrac{\pi x}{6}$

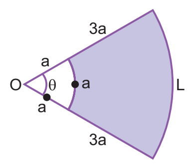
$2x - 1 = \dfrac{x}{2}$

$\dfrac{3x}{2} = 1$

$\therefore\ x = \dfrac{2}{3}$

Clave C

8.



Del gráfico:

$\theta \cdot a = a$

$\Rightarrow \theta = 1$ rad

También: $L = \theta \cdot 4a$

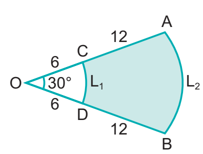
$L = (1)4a \Rightarrow L = 4a$

Piden el perímetro de la región sombreada:

$a + 3a + 3a + L = 7a + (4a) = 11a$

Clave D

9.



$30° = 30°\cdot\left(\dfrac{\pi\text{ rad}}{180°}\right) = \dfrac{\pi}{6}$ rad

Se cumple:

$L_1 = \left(\dfrac{\pi}{6}\right)(6) = \pi$

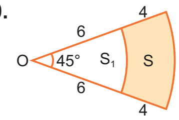
$L_2 = \left(\dfrac{\pi}{6}\right)(18) = 3\pi$

Piden el perímetro de la región sombreada:

$L_1 + 12 + 12 + L_2 = \pi + 24 + 3\pi = 4(\pi + 6)$

Clave E

10.



$45°\cdot\left(\dfrac{\pi\text{ rad}}{180°}\right) = \dfrac{\pi}{4}$

$\Rightarrow 45° = \dfrac{\pi}{4}$ rad

Del gráfico:

$S_1 + S = \dfrac{\left(\frac{\pi}{4}\right)(10)^2}{2}$

$S_1 + S = \dfrac{25\pi}{2}$

$\dfrac{\frac{\pi}{4}(6)^2}{2} + S = \dfrac{25\pi}{2}$

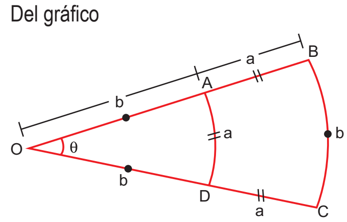
$\dfrac{9\pi}{2} + S = \dfrac{25\pi}{2}$

$S = \dfrac{16\pi}{2}$

$\therefore\ S = 8\pi$

Clave E

11. Del gráfico



Del sector AOD:

$a = \theta b \Rightarrow \theta = \dfrac{a}{b}$ ... (1)

Por propiedad del trapecio ABCD:

$\theta = \dfrac{b-a}{a} = \dfrac{b}{a} - 1 \Rightarrow \theta = \dfrac{b}{a} - 1$ ... (2)

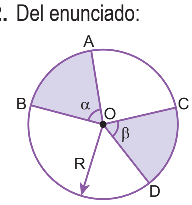
De (1) y (2):

$\theta = \theta^{-1} - 1$

$\therefore\ \theta^{-1} - \theta = 1$

Clave B

12. Del enunciado:



Sean α y β los ángulos complementarios:

$\alpha + \beta = \dfrac{\pi}{2}$ ... (1)

Además:

$L_{\overset{\frown}{AB}} = \alpha R$; $L_{\overset{\frown}{CD}} = \beta R$

Del dato:

$L_{\overset{\frown}{AB}} + L_{\overset{\frown}{CD}} = 5\pi$ cm, reemplazamos:

$\alpha R + \beta R = 5\pi$

$R(\alpha + \beta) = 5\pi$

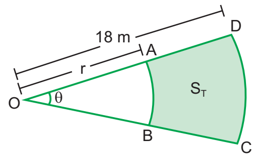
De (1):

$R\left(\dfrac{\cancel{\pi}}{2}\right) = 5\cancel{\pi} \Rightarrow R = 10$

$\therefore\ R = 10$ cm

Clave C

13. Del enunciado:



Sea S longitud de una circunferencia; entonces

$L_{\overset{\frown}{AB}} = \dfrac{S}{6}$; donde $S = 2\pi r$, longitud de la circunferencia de radio r.

$L_{\overset{\frown}{AB}} = \dfrac{2\pi r}{6}$

$\theta r = \dfrac{2\pi r}{6}$

$\theta = \dfrac{\pi}{3}$

El área del trapecio será:

$S_T = \dfrac{1}{2}\theta(18)^2 - S_{\triangleleft AOB}$

Por dato: $S_{\triangleleft AOB} = 24\pi\ m^2$

Reemplazamos:

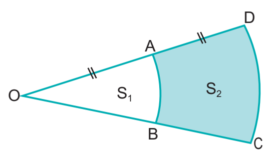
$S_T = \dfrac{1}{2}\left(\dfrac{\pi}{3}\right)(18)^2 - 24\pi$

$S_T = 54\pi - 24\pi$

$\therefore\ S_T = 30\pi\ m^2$

Clave E

14.



En los sectores circulares, por propiedad se tiene que:

$S_2 = 3S_1$

Entonces:

$3S_{\triangleleft AOB} = S$; por dato: $S = 63\ cm^2$

$3S_{\triangleleft AOB} = 63$

$S_{\triangleleft AOB} = 21\ cm^2$

Clave D

PRACTIQUEMOS

Nivel 1 (página 13) Unidad 1

Comunicación matemática

1. Calculamos las áreas de las cuatro regiones sombreadas:

$S_1 = \dfrac{1}{2}(2\theta)R^2 = 2\left(\dfrac{1}{2}\theta R^2\right)$ ... (1)

$S_2 = \dfrac{1}{2}\theta R^2 = 1\left(\dfrac{1}{2}\theta R^2\right)$ ... (2)

$S_3 = \dfrac{1}{2}\cdot\dfrac{\theta}{2}(3R)^2 = \dfrac{9}{2}\left(\dfrac{1}{2}\theta R^2\right)$ ... (3)

$S_4 = \dfrac{1}{2}\cdot 4\theta\,(3R)^2 = 36\left(\dfrac{1}{2}\theta R^2\right)$ ... (4)

I. De (1) y (2):

$\dfrac{S_1}{S_2} = \dfrac{2\left(\frac{1}{2}\theta R^2\right)}{1\left(\frac{1}{2}\theta R^2\right)} \Rightarrow \dfrac{S_2}{S_1} = \dfrac{1}{2}$

II. De (2) y (3):

$\dfrac{S_3}{S_2} = \dfrac{\frac{9}{2}\left(\frac{1}{2}\theta R^2\right)}{1\left(\frac{1}{2}\theta R^2\right)} \Rightarrow \dfrac{S_3}{S_2} = \dfrac{9}{2}$

III. De (1) y (4):

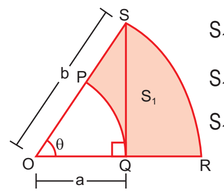
$\dfrac{S_1}{S_4} = \dfrac{2\left(\frac{1}{2}\theta R^2\right)}{36\left(\frac{1}{2}\theta R^2\right)} \Rightarrow \dfrac{S_1}{S_4} = \dfrac{1}{18}$

Entonces:

Id; IIc; IIIa

Clave C

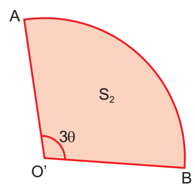
2. De la figura 1:



$S_1 = \dfrac{1}{2}\theta b^2 - \dfrac{1}{2}\theta a^2$

$S_1 = \dfrac{1}{2}\theta(b^2 - a^2)$

$S_1 = \dfrac{1}{2}\,\theta y^2$ ... (1)



Del dato:

$O'B = SQ = y$

$S_2 = \dfrac{1}{2}\,.\,3\theta\,.\,y^2$

$S_2 = \dfrac{3}{2}\,\theta y^2$ ... (2)

De (1) y (2):

$S_2 = 3\left(\dfrac{1}{2}\theta y^2\right) = 3S_1$

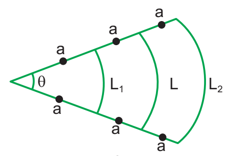
$\dfrac{S_2}{S_1} = \dfrac{3}{1} \Rightarrow S_1 < S_2$

$\therefore$ S_1 es a S_2 como 1 es a 3.

Clave E

Razonamiento y demostración

3.



$L_1 = \theta\ .\ a$

$L = \theta\ .\ 2a$

$L_2 = \theta\ .\ 3a$

$L_1 + L_2 = 8\pi$

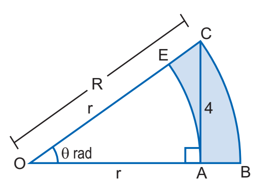
$\theta a + 3\theta a = 8\pi$

$4\theta a = 8\pi \quad \Rightarrow \quad \theta a = 2\pi$

$\therefore\ L = \theta\ .\ 2a = 2(\theta a) = 4\pi$

Clave D

4.



Piden el área de la región sombreada.

$\Rightarrow A_{somb.} = \dfrac{\theta.R^2}{2} - \dfrac{\theta.r^2}{2}$

$\Rightarrow A_{somb.} = \dfrac{\theta}{2}(R^2 - r^2)$ ...(1)

En el ⊿OAC por el teorema de Pitágoras:

$R^2 = 4^2 + r^2 \Rightarrow R^2 - r^2 = 16$ ...(2)

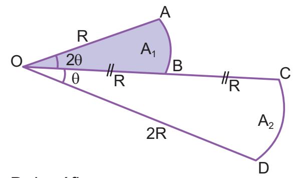
Reemplazando (2) en (1):

$\Rightarrow A_{somb.} = \dfrac{\theta}{2}(16) = 8\theta$

$\therefore\ A_{somb.} = 8\theta$

Clave D

5.



Del gráfico:

$A_1 = \dfrac{(2\theta).R^2}{2} = \theta R^2$

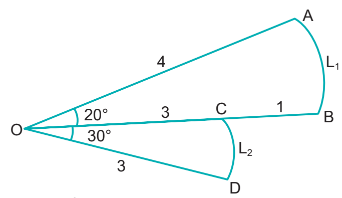
$A_2 = \dfrac{\theta.(2R)^2}{2} = 2\theta R^2$

Piden:

$J = \dfrac{A_1}{A_2} = \dfrac{\theta R^2}{2\theta R^2} = \dfrac{1}{2}$ $\qquad \therefore\ J = \dfrac{1}{2}$

Clave B

6.



Del gráfico:

$L_1 = \left(20°\,.\,\dfrac{\pi\ rad}{180°}\right)(4) = \dfrac{4\pi}{9}$

$L_2 = \left(30°\,.\,\dfrac{\pi\ rad}{180°}\right)(3) = \dfrac{\pi}{2}$

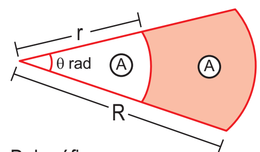
Piden:

$J = \dfrac{L_1}{L_2} = \dfrac{\frac{4\pi}{9}}{\frac{\pi}{2}} = \dfrac{8}{9}$

$\therefore\ J = \dfrac{8}{9}$

Clave B

7.



Del gráfico:

$A = \dfrac{\theta.r^2}{2} = \dfrac{\theta r^2}{2}$ ...(1)

$2A = \dfrac{\theta.R^2}{2} = \dfrac{\theta R^2}{2}$ ...(2)

Dividiendo (2) y (1):

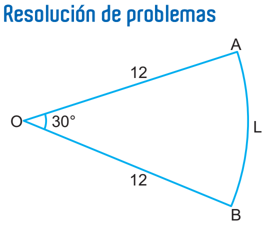
$\dfrac{R^2}{r^2} = \dfrac{2A}{A} \Rightarrow \dfrac{R^2}{r^2} = \dfrac{2}{1}$

$\therefore\ \dfrac{R}{r} = \sqrt{2}$

Clave D

Resolución de problemas

8.



Se tiene: $30° = \frac{\pi}{6}$ rad

Luego: $L = \theta \,.\, R$

$$L = \left(\frac{\pi}{6}\right)(12) = 2\pi$$

Piden: el perímetro del sector (2p).

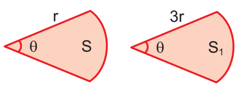
$2p = 12 + 12 + L$

$\Rightarrow 2p = 24 + (2\pi) = 2(12 + \pi)$

$\therefore\ 2p = 2(12 + \pi)$ cm

Clave C

9.



$S = \frac{\theta \,.\, r^2}{2}$ $\qquad S_1 = \frac{\theta \,.\, (3r)^2}{2}$

$S_1 = 9\frac{(\theta \,.\, r^2)}{2}$

$\therefore\ S_1 = 9S$

Clave E

10. Por dato:

$A = 2\pi$ cm^2

$L = \pi$ cm

Piden la medida del radio (R).

Se cumple: $A = \frac{L.R}{2}$

$\Rightarrow 2\pi = \frac{\pi(R)}{2} \Rightarrow R = 4 \qquad \therefore\ R = 4$ cm

Clave D

11. Por dato:

$L = 2\pi$ cm

$R = 12$ cm

Piden el área del sector circular (A).

$A = \frac{L.R}{2} = \frac{(2\pi)(12)}{2} = 12\pi$

$\therefore\ A = 12\pi$ cm^2

Clave B

12. Por dato:

$L = 2\pi$ cm

$\theta = 40° \,.\, \left(\frac{\pi \text{ rad}}{180°}\right) = \frac{2\pi}{9}$ rad

Piden el área del sector circular (A).

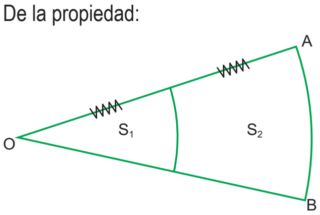
$A = \frac{L^2}{2\theta} = \frac{(2\pi)^2}{2\left(\frac{2\pi}{9}\right)} = 9\pi \qquad \therefore\ A = 9\pi$ cm^2

Clave A

Nivel 2 (página 14) Unidad 1

Comunicación matemática

13. De la propiedad:



$S_2 = 3S_1$

Entonces:

I. $3S_{\sphericalangle AOP} = S_1 \Rightarrow S_{\sphericalangle AOP} = \frac{S_1}{3}$ ... (1)

$3S_{\sphericalangle POB} = S_2 \Rightarrow S_{\sphericalangle POB} = \frac{S_2}{3}$ ... (2)

Por dato:

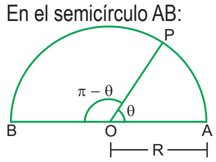
$5S_{\sphericalangle AOP} = S_{\sphericalangle POB}$

De (1) y (2):

$5\frac{S_1}{\cancel{3}} = \frac{S_2}{\cancel{3}} \Rightarrow \frac{S_1}{S_2} = \frac{1}{5}$

$\therefore\ S_1$ es a S_2 como 1 es a 5.

II. En el semicírculo AB:



$S_{\sphericalangle AOP} = \frac{1}{2}\theta R^2; \quad S_{\sphericalangle POB} = \frac{1}{2}(\pi - \theta)R^2$

Por dato:

$5S_{\sphericalangle AOP} = S_{\sphericalangle POB}$

$\Rightarrow 5\ \frac{1}{2}\theta R^2 = \frac{1}{2}(\pi - \theta)R^2$

Luego:

$5\theta = \pi - \theta$

$6\theta = \pi$

$\theta = \frac{\pi}{6}$

$\therefore\ \theta$ es igual a $\frac{\pi}{6}$ rad.

III. Si R = 6 m se tiene:

$S_{\sphericalangle AOP} = \frac{1}{2}\theta R^2 = \frac{1}{2}\theta(6)^2 = 18\theta$

De II; $\theta = \frac{\pi}{6}$ rad

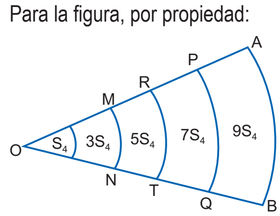
$S_{\sphericalangle AOP} = 18\theta = 18 \,.\, \frac{\pi}{6} = 3\pi$

$\therefore\ S_{\sphericalangle AOP} = 3\pi$ m^2

$\therefore$ F V V

Clave C

14. Para la figura, por propiedad:



Luego:

$S_{\sphericalangle AOB} = 25S_4$

Por dato:

$S_{\sphericalangle AOB} = 25S = 25S_4 \Rightarrow S_4 = S$

Además:

- $S_1 = 4S_4 = 4S$
- $S_2 = 5S_4 = 5S$
- $S_3 = 16S_4 = 16S$

I. $\frac{S_1 + S_3}{5} = \frac{4S + 16S}{5} = \frac{\cancel{20}S}{\cancel{5}} = 4S$

$\frac{S_1 + S_3}{5} = 4S$

II. $4S_2 = 4 \,.\, 5S = 20S$

$\therefore\ 4S_2 = 20S$

III. $\frac{S_3 - S_2}{11} = \frac{16S - 5S}{11} = \frac{11S}{11} = S$

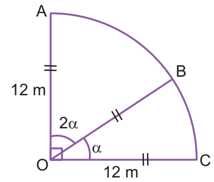
$\therefore\ \frac{S_3 - S_2}{11} = S$

Entonces: Ic, IIa, IIIb

Clave D

Razonamiento y demostración

15. Del gráfico:



Piden: $L_{\widehat{AB}}$

$L_{\widehat{AB}} = 2\alpha(12$ m$)$

Pero: $3\alpha = 90° = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{6}$

Reemplazando:

$L_{\widehat{AB}} = \frac{2\pi}{6}(12$ m$) = 4\pi$ m

Clave D

16. Del gráfico:

$6\alpha = 180°$

$\alpha = 30° \times \frac{\pi}{180°} = \frac{\pi}{6}$ rad

Piden:

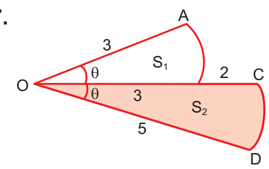
$L_{\widehat{BC}} = 24 \times 2\alpha$

$L_{\widehat{BC}} = 24 \times 2\left(\frac{\pi}{6}\right)$

$L_{\widehat{BC}} = 8\pi$ m

Clave C

17.



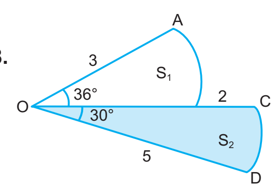
$S_1 = \frac{\theta \,.\, r^2}{2} = \frac{\theta \,.\, 3^2}{2} = \frac{9\theta}{2}$

$S_2 = \frac{\theta \,.\, r^2}{2} = \frac{\theta \,.\, 5^2}{2} = \frac{25\theta}{2}$

$\frac{S_1}{S_2} = \frac{\frac{9\theta}{2}}{\frac{25\theta}{2}} = \frac{9}{25} = 0{,}36$

Clave C

18.



36° a radianes:

$\frac{S}{9} = \frac{20R}{\pi} \Rightarrow \frac{36}{9} = \frac{20R}{\pi}$

$R = \frac{\pi}{5} \Rightarrow \theta_1 = \frac{\pi}{5}$ rad

Luego: $S_1 = \frac{\theta_1 \,.\, r^2}{2} = \frac{\pi}{5} \,.\, \frac{3^2}{2}$

$S_1 = \frac{9}{10}\pi$

30° a radianes:

$\frac{S}{9} = \frac{20R}{\pi} \Rightarrow \frac{30}{9} = \frac{20R}{\pi}$

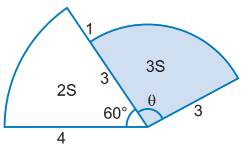
$\Rightarrow R = \frac{\pi}{6} \Rightarrow \theta_2 = \frac{\pi}{6}$ rad

Luego: $S_2 = \frac{\theta_2 \times r^2}{2} = \frac{\pi}{6} \times \frac{5^2}{2} = \frac{25\pi}{12}$

$\Rightarrow \frac{S_1}{S_2} = \frac{\frac{9\pi}{10}}{\frac{25\pi}{12}} = \frac{9 \times 12}{10 \times 25} = \frac{54}{125} = 0{,}432$

Clave D

19.



60° a radianes:

$\frac{S}{9} = \frac{20R}{\pi}$

$\frac{60}{9} = \frac{20R}{\pi}$

$R = \frac{\pi}{3} \Rightarrow \theta_2 = \frac{\pi}{3}$ rad

Luego: $2S = \frac{\pi}{3}\frac{(4)^2}{2}$

$2S = \frac{8\pi}{3} \Rightarrow S = \frac{4\pi}{3}$

Además:

$3S = \frac{\theta \, . \, 3^2}{2}$

$3\left(\frac{4\pi}{3}\right) = \frac{9}{2} \, . \, \theta \quad \Rightarrow \quad \theta = \frac{8\pi}{9}$ rad

θ a sexagesimal:

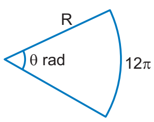
$\frac{S}{9} = \frac{20}{\pi}\left(\frac{8}{9}\pi\right)$

$\theta = 160°$

Clave E

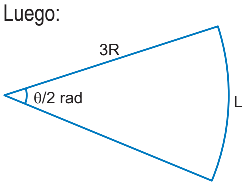
Resolución de problemas

20. Del enunciado:



$\Rightarrow \theta \, . \, R = 12\pi \qquad ...(1)$

Luego:



$\Rightarrow \left(\frac{\theta}{2}\right)(3R) = L$

$\Rightarrow L = \frac{3}{2}(\theta \, . \, R) \qquad ...(2)$

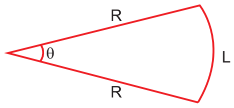
Reemplazando (1) en (2):

$\Rightarrow L = \frac{3}{2}(12\pi) = 3(6\pi)$

$\therefore \; L = 18\pi$ cm

Clave B

21.



Por dato: L = 3R

$\underbrace{\text{Perímetro}}_{} = 30$

$2R + L = 30$

$2R + 3R = 30 \Rightarrow 5R = 30$

$R = 6$

Si: $R = 6 \Rightarrow L = 18$

Área del sector circular: $S = \frac{L.R}{2} = \frac{18 \, . \, 6}{2}$

$\Rightarrow S = 54 \text{ m}^2$

Clave D

22. Se sabe: $S = \frac{L^2}{2\theta}$

Pero, por datos:

$\theta = 0{,}785$ rad

$L = 6{,}28$ m

Reemplazando:

$S = \frac{6{,}28 \times 6{,}28}{2(0{,}785)}$

$S = \frac{6{,}28 \times 2 \times 3{,}14}{1{,}57}$

$S = \frac{6{,}28 \times 2 \times \pi}{1{,}57}$

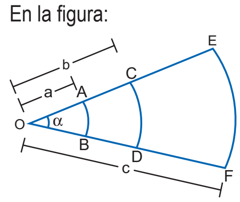
$S = 8\pi \text{ m}^2$

Clave C

Nivel 3 (página 15) Unidad 1

Comunicación matemática

23. En la figura:



Luego:

$S_{\sphericalangle AOB} = \frac{1}{2}\alpha a^2; \quad S_{\sphericalangle COD} = \frac{1}{2}\alpha b^2$

$S_{\sphericalangle EOF} = \frac{1}{2}\alpha c^2$

De la conclusión:

$S_{\sphericalangle AOB} = \frac{S_{\sphericalangle COD}}{4}$; $4S_{\sphericalangle AOB} = S_{\sphericalangle COD}$

Reemplazando:

$4\left(\frac{1}{2}\cancel{\alpha} a^2\right) = \frac{1}{2}\cancel{\alpha} b^2$

$4a^2 = b^2$

$2a = b$ ... (1)

Además:

$S_{\sphericalangle AOB} = \frac{S_{\sphericalangle EOF}}{16}$; $16S_{\sphericalangle AOB} = S_{\sphericalangle EOF}$

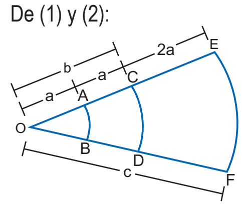
Reemplazando:

$16\left(\frac{1}{2}\alpha a^2\right) = \frac{1}{2}\alpha c^2$

$16a^2 = c^2$

$4a = c \quad$ ... (2)

De (1) y (2):



Luego:

I. A es punto medio de $\overline{OC}$.

(Falsa)

II. AC = a, AE = 3a, es decir:

$\frac{AC}{AE} = \frac{a}{3a} = \frac{1}{3}$

De las proposiciones, AC es a AE como 1 es a 3.

(Verdadera)

III. De la figura:

AC = a, CE = 2a

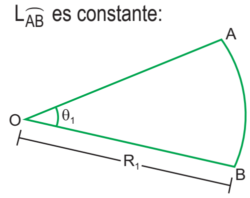
(Falsa)

Clave D

24. De la figura se tiene que:

$L_{\widehat{AB}} = \theta_i R_i \qquad ... (1)$

I. Sea el nuevo sector circular donde $L_{\widehat{AB}}$ es constante:



El radio disminuye a la mitad; entonces: $R_1 = \frac{R_i}{2}$

Además:

$\theta_1 \, . \, R_1 = L_{\widehat{AB}}$

De (1):

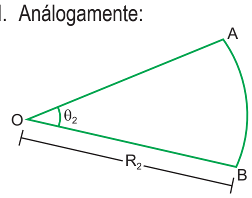
$\theta_1 \, . \, R_1 = \theta_i R_i$; reemplazamos R_1:

$\theta_1 \, . \, \frac{R_i}{2} = \theta_i R_i$

$\theta_1 = 2\theta_i$ (El ángulo se duplica)

$\therefore$ El radio disminuye a la mitad entonces el ángulo se duplica.

II. Análogamente:



El ángulo disminuye a 3/4 de su valor inicial:

$\theta_2 = \frac{3}{4}\theta_i$

Además:

$R_2\theta_2 = L_{\widehat{AB}}$

De (1):

$R_2\theta_2 = R_i\theta_i$; reemplazamos θ_2:

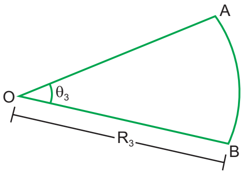
$R_2 \frac{3}{4}\theta_i = R_i\theta_i$

$R_2 = \frac{4}{3}R_i$

$R_2 = R_i + \frac{1}{3}R_i$ (El radio aumenta 1/3 de su valor inicial)

∴ El ángulo disminuye a 3/4 de su valor inicial, entonces el radio aumenta a 1/3 de su valor inicial.

III. Finalmente:



El ángulo se incrementa en 2/3 de su valor:

$\theta_3 = \theta_i + \frac{2}{3}\theta_i$

$\theta_3 = \frac{5}{3}\theta_i$

Además:

$\theta_3 R_3 = L_{\overset{\frown}{AB}}$

De (1):

$\theta_3 R_3 = \theta_i R_i$, reemplazamos θ_3:

$\frac{5}{3}\theta_i R_3 = \theta_i R_i$

$R_3 = \frac{3}{5} R_i$

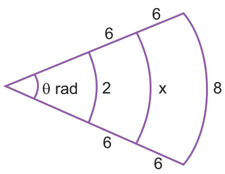
$R_3 = R_i - \frac{2}{5} R_i$ (El radio disminuye en 2/5 de su valor inicial)

∴ El ángulo se incrementa en 2/3 de su valor, entonces el radio disminuye en 2/5 de su valor inicial.

Clave B

Razonamiento y demostración

25.



Por la propiedad:

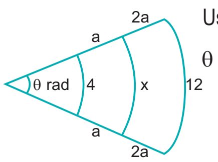
$\theta = \frac{8-2}{12} = \frac{x-2}{6}$

$\frac{6}{12} = \frac{x-2}{6}$

$\Rightarrow x = 5$

Clave A

26.



Usamos la propiedad:

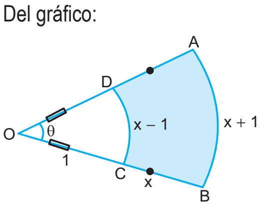
$\theta = \frac{12-4}{3a} = \frac{x-4}{a}$

$\frac{8}{3} = x - 4$

$\Rightarrow x = \frac{20}{3}$

Clave C

27. Del gráfico:



Piden:

$A_{somb.} = \frac{(x-1+x+1)x}{2}$

$A_{somb.} = x^2$

Pero:

$L_{\overset{\frown}{DC}} = x - 1 = \theta(1)$

$\Rightarrow \theta = x - 1$

$L_{\overset{\frown}{AB}} = x + 1 = \theta(x+1)$

$x + 1 = (x-1)(x+1)$

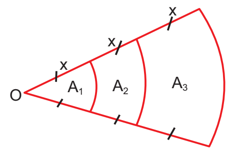
$\Rightarrow x = 2$

Reemplazando:

$A_{somb.} = (2)^2 = 4$

Clave C

28. Del gráfico:



Por teoría: $A_2 = 3A_1$; $A_3 = 5A_1$

Piden:

$E = \frac{A_1A_3 + A_3A_2 - A_2A_1}{(A_1)^2 - (A_2)^2 + (A_3)^2}$

Entonces:

$E = \frac{A_1(5A_1) + (5A_1)(3A_1) - (3A_1)A_1}{(A_1)^2 - (3A_1)^2 + (5A_1)^2}$

$E = \frac{5A_1^2 + 15A_1^2 - 3A_1^2}{A_1^2 - 9A_1^2 + 25A_1^2}$

$E = \frac{17A_1^2}{17A_1^2} = 1$

Clave A

Resolución de problemas

29. $S_1 = 100\ m^2$

R: radio

L: longitud de arco

$S_1 = \frac{LR}{2}$

Reemplazando:

$100 = \frac{LR}{2}$ ...(1)

$S_2 = ?$

$R_2 = 120\%R$

$L_2 = 60\%L$

Reemplazando:

$S_2 = \frac{(120\%R)(60\%L)}{2}$ ...(2)

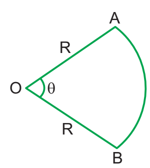
(2) ÷ (1):

$\frac{S_2}{100} = \frac{(120\%R)(60\%L)}{LR}$

Simplificamos: $S_2 = 72\ m^2$

Clave B

30. Sea el sector circular:



Perímetro $= 10 = 2R + L_{\overset{\frown}{AB}}$

$\Rightarrow L_{\overset{\frown}{AB}} = 10 - 2R$

Área $= 6 = \frac{L_{\overset{\frown}{AB}}R}{2} = \frac{(10-2R)R}{2}$

$12 = 10R - 2R^2$

$2R^2 - 10R + 12 = 0$

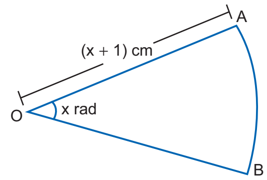
$2R \quad -4 \Rightarrow R = 2 \Rightarrow \theta = 3$

$R \quad -3 \Rightarrow R = 3 \Rightarrow \theta = 2$

Por lo tanto, hay dos respuestas.

Clave E

31. Del enunciado:



$S_{\sphericalangle AOB} = \frac{1}{2}x(x+1)^2$ ... (1)

Por dato:

$S_{\sphericalangle AOB} = x$ ... (2)

Entonces:

$\cancel{x} = \frac{1}{2}\cancel{x}(x+1)^2$

$2 = (x+1)^2$; $x > 0$

$\sqrt{2} = x + 1$

$x = \sqrt{2} - 1$

$L_{\overset{\frown}{AB}} = \theta.R = x(x+1)$

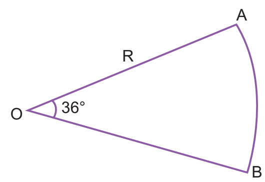
$L_{\overset{\frown}{AB}} = (\sqrt{2}-1)(\sqrt{2}-1+1) = (\sqrt{2}-1)\sqrt{2}$

$L_{\overset{\frown}{AB}} = 2 - \sqrt{2}$

$\therefore\ L_{\overset{\frown}{AB}} = (2 - \sqrt{2})$ cm

Clave D

32. Del enunciado:



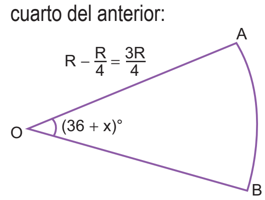
Luego:

$36° = 36\cancel{0}.\frac{\pi}{180\cancel{0}}\text{rad} = \frac{\pi}{5}\text{ rad}$

$S_{\sphericalangle AOB} = \frac{1}{2}\left(\frac{\pi}{5}\right).R^2 = \frac{\pi R^2}{10}$

$S_{\sphericalangle AOB} = \frac{\pi R^2}{10}$ ... (1)

El ángulo aumenta en x° y el radio disminuye un cuarto del anterior:



$(36+x)° = (36+x)\cancel{°}.\frac{\pi}{180\cancel{°}}\text{rad} = \frac{(36+x)}{180}\pi\text{ rad}$

$S_{\sphericalangle AOB} = \frac{1}{2}.\frac{(36+x)\pi}{180}.\left(\frac{3}{4}R\right)^2$ ... (2)

El área del sector no varía; de (1) y (2):

$S_{\sphericalangle AOB} = \frac{\pi R^2}{10} = \frac{1}{2}.\frac{(36+x)\pi}{180}\left(\frac{3}{4}\right)^2R^2$

$1 = \frac{(36+x)}{64}$

$64 = 36 + x$

$x = 28$

∴ El ángulo central aumenta en 28°.

Clave A

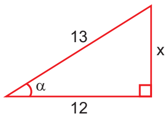
APLICAMOS LO APRENDIDO (página 16) Unidad 1

1. Por el teorema de Pitágoras:

$13^2 = 12^2 + x^2$

$25 = x^2$

$x = 5$

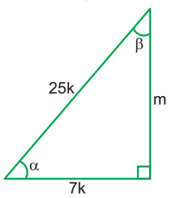


Piden:

$\cot\alpha = \dfrac{12}{x} = \dfrac{12}{5} \qquad \Rightarrow \cot\alpha = \dfrac{12}{5}$

Clave A

2. Sea: $\alpha > \beta$



Por dato:

$\sec\alpha = \dfrac{25}{7}$

Por el teorema de Pitágoras:

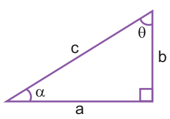
$m = 24k$

Piden:

$\cot\beta = \dfrac{m}{7k} = \dfrac{24k}{7k} = \dfrac{24}{7}$

Clave C

3.



Por dato:

$\dfrac{a}{b} = \dfrac{18}{24} = \dfrac{3}{4}$

$\Rightarrow a = 3k \ \wedge\ b = 4k$

Por el teorema de Pitágoras:

$c = 5k$

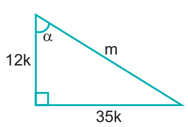
Como: $b > a \Rightarrow \alpha > \theta$

Piden: $\cos\alpha$

$\cos\alpha = \dfrac{a}{c} = \dfrac{3k}{5k} = \dfrac{3}{5} \qquad \Rightarrow \cos\alpha = \dfrac{3}{5}$

Clave E

4.



Por dato:

$\tan\alpha = \dfrac{35}{12}$

Por el teorema de Pitágoras:

$m = 37k$

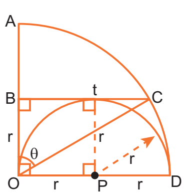
Piden: $\csc\alpha$

$\csc\alpha = \dfrac{m}{35k} = \dfrac{37k}{35k} = \dfrac{37}{35}$

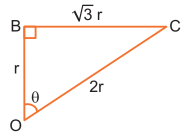
$\therefore\ \csc\alpha = \dfrac{37}{35}$

Clave D

5.



Del gráfico: $OC = OD = 2r$



Por el teorema de Pitágoras:

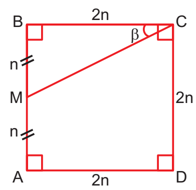
$BC = \sqrt{3}\,r$

Piden:

$\operatorname{sen}\theta = \dfrac{\sqrt{3}\,r}{2r} = \dfrac{\sqrt{3}}{2} \quad \Rightarrow \quad \operatorname{sen}\theta = \dfrac{\sqrt{3}}{2}$

Clave C

6.

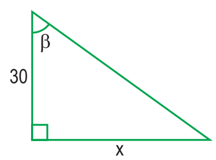


Piden:

$\tan\beta = \dfrac{BM}{BC} = \dfrac{n}{2n} = \dfrac{1}{2} \qquad \therefore\ \tan\beta = \dfrac{1}{2}$

Clave E

7.



Por dato:

$\cot\beta = \dfrac{5}{12}$

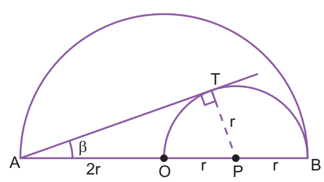
Del gráfico:

$\cot\beta = \dfrac{30}{x}$

Entonces: $\dfrac{30}{x} = \dfrac{5}{12} \Rightarrow x = 72$

Clave C

8.



Del gráfico: T punto de tangencia.

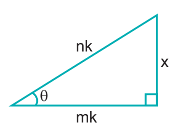
Piden: $\operatorname{sen}\beta$

$\operatorname{sen}\beta = \dfrac{PT}{PA} = \dfrac{r}{3r} = \dfrac{1}{3}$

$\therefore\ \operatorname{sen}\beta = \dfrac{1}{3}$

Clave C

9.



Por dato:

$\cos\theta = \dfrac{m}{n}$

Por el teorema de Pitágoras:

$x = k\sqrt{n^2 - m^2}$

Piden:

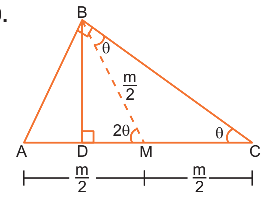
$P = \sqrt{n^2 - m^2}\,.\cot\theta$

$P = \sqrt{n^2 - m^2}\,.\dfrac{mk}{k\sqrt{n^2 - m^2}}$

$\therefore\ P = m$

Clave D

10.



Trazamos la mediana BM.

Por propiedad: $BM = AM = MC$

En el ⊿BDM:

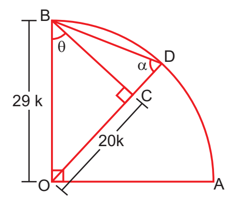
$\operatorname{sen}2\theta = \dfrac{BD}{BM} \Rightarrow BD = BM\,.\operatorname{sen}2\theta$

$BD = \left(\dfrac{m}{2}\right)\operatorname{sen}2\theta$

$\therefore\ BD = \dfrac{m\operatorname{sen}2\theta}{2}$

Clave E

11. En el triángulo OCB:



$\operatorname{sen}\theta = \dfrac{CO}{BO} = \dfrac{20}{29}$

$\Rightarrow BO = 29k,$

$CO = 20k$

Donde:

BO: radio del sector circular.

En el ⊿BCD:

$CD = OD - OC = 29k - 20k$

$CD = 9k \qquad \ldots (1)$

Por Pitágoras en el ⊿BCO:

$OC^2 + BC^2 = BO^2$

$(20k)^2 + BC^2 = (29k)^2$

$BC^2 = (29k)^2 - (20k)^2$

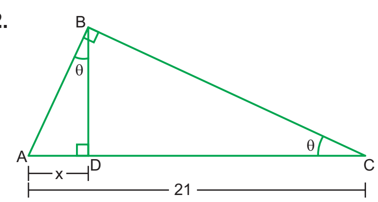
$BC = 21k \qquad \ldots (2)$

De (1) y (2):

$\tan\alpha = \dfrac{BC}{CD} = \dfrac{21k}{9k} \qquad \therefore\ \tan\alpha = \dfrac{7}{3}$

Clave B

12.



En el ⊿ABC:

$\operatorname{sen}\theta = \dfrac{AB}{AC} = \dfrac{3}{7}$

Por dato: $AC = 21$

$\Rightarrow \dfrac{AB}{21} = \dfrac{3}{7}$

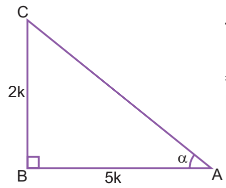
$AB = 9$

En el ⊿BDA:

$\operatorname{sen}\theta = \dfrac{AD}{AB} = \dfrac{x}{9} = \dfrac{3}{7} \qquad \therefore\ x = \dfrac{27}{7}$

Clave A

13. Sea el triángulo rectángulo ABC, donde: $\tan\alpha = \frac{2}{5}$



$\tan\alpha = \frac{BC}{AB} = \frac{2}{5}$

$\Rightarrow BC = 2k \wedge AB = 5k$

Por dato:

$5k + 2k = 21$

$7k = 21$

$k = 3$

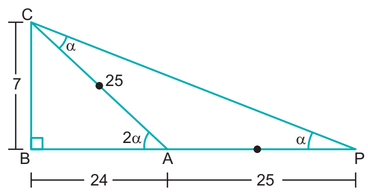
Nos piden:

$AB - BC = 5k - 2k = 3k = 3(3)$

$\therefore\ AB - BC = 9$

Clave B

14. Sea el triángulo rectángulo ABC:



Por dato:

$\sec A = \frac{25}{24}$

Sea: $AC = 25 \Rightarrow AB = 24$

Por T. de Pitágoras:

$AB^2 + BC^2 = AC^2$

$24^2 + BC^2 = (25)^2$

$BC^2 = (25)^2 - (24)^2$

$BC = 7$

Del triángulo CAP, isósceles:

$AP = AC = 25$

Nos piden:

$\tan\frac{A}{2} = \tan\alpha = \frac{BC}{BP}$

De la figura:

$\frac{BC}{BP} = \frac{7}{24 + 25} = \frac{7}{49}$ $\quad\therefore\ \tan\frac{A}{2} = \frac{1}{7}$

Clave E

PRACTIQUEMOS

Nivel 1 (página 18) Unidad 1

Comunicación matemática

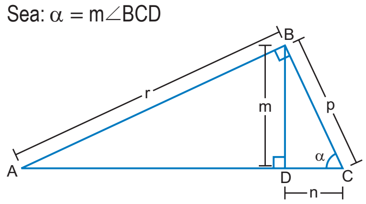
1. I. Por dato en el triángulo BDC se cumple:

$m^2 + n^2 = p^2$

Entonces:

El triángulo BDC es un triángulo rectángulo recto en D. (V)

II. Sea: $\alpha = m\angle BCD$



Del ◺ABC y ◺BDC:

$\tan\alpha = \frac{r}{p} = \frac{m}{n}$

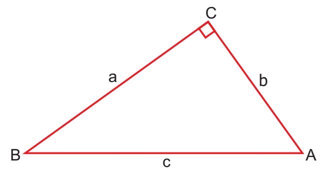
$\therefore\ \frac{r}{p}$ y $\frac{m}{n}$ son equivalentes. (V)

III. Sabemos que: $m\angle ADB = 90°$, entonces $\overline{BD}$ es la altura relativa al lado $\overline{AC}$ en el triángulo ABC. (V)

Clave E

2. I. Se cumple: $a^2 + b^2 = c^2$ (teorema de Pitágoras) por lo tanto, ABC es un triángulo rectángulo. (V)

II. De I, si: $a^2 + b^2 = c^2$, entonces:



$\Rightarrow$ A y B son ángulos agudos. (F)

III. De II:

$\Rightarrow$ C es un ángulo recto. (F)

Clave B

Razonamiento y demostración

3. Del gráfico:

En el ◺ ABC:

$\tan\theta = \frac{CB}{AB}$ ...(1)

En el ◺ MBC:

$\tan\theta = \frac{MB}{CB}$

Pero: $MB = \frac{1}{2}AB$

Entonces: $\tan\theta = \frac{\frac{1}{2}AB}{CB}$ ...(2)

Igualando (1) y (2):

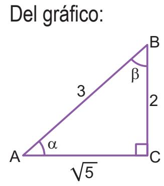
$\frac{CB}{AB} = \frac{AB}{2CB}$

$\frac{CB^2}{AB^2} = \frac{1}{2} \Rightarrow \frac{CB}{AB} = \frac{1}{\sqrt{2}} = \tan\theta$

$\therefore\ \tan\theta = \frac{\sqrt{2}}{2}$

Clave E

4. Del gráfico:



Piden:

$M = \frac{\operatorname{sen}\alpha + \operatorname{sen}\beta}{\cos\beta} + \cot\alpha$

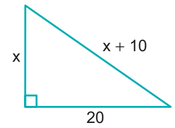
$M = \frac{\frac{2}{3} + \frac{\sqrt{5}}{3}}{\frac{2}{3}} + \frac{\sqrt{5}}{2}$

$M = \frac{2 + \sqrt{5}}{2} + \frac{\sqrt{5}}{2}$

$M = \frac{2 + 2\sqrt{5}}{2} = 1 + \sqrt{5}$

Clave A

5.



$x^2 + (20)^2 = (x + 10)^2$

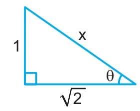
$x^2 + 400 = x^2 + 20x + 100$

$300 = 20x$

$\therefore x = 15$

Clave C

6. $\tan\theta = \frac{1}{\sqrt{2}} = \frac{CO}{CA}$



$x^2 = 1^2 + \sqrt{2}^2$

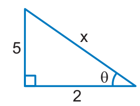
$x^2 = 3$

$x = \sqrt{3}$

$\therefore\ \operatorname{sen}\theta = \frac{CO}{H} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Clave B

7. $\cot\theta = 0{,}4 = \frac{4}{10} = \frac{2}{5} = \frac{CA}{CO}$



$x^2 = 5^2 + 2^2$

$x^2 = 29$

$x = \sqrt{29}$

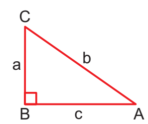
$E = \sec\theta\csc\theta$

$E = \frac{H}{CA} \cdot \frac{H}{CO} = \frac{\sqrt{29}}{2} \cdot \frac{\sqrt{29}}{5}$

$\therefore\ E = \frac{29}{10} = 2{,}9$

Clave A

8.



$\tan A = \frac{a}{c}$ $\qquad \cot A = \frac{c}{a}$

$\sec C = \frac{b}{a}$ $\qquad \csc C = \frac{b}{c}$

Además:

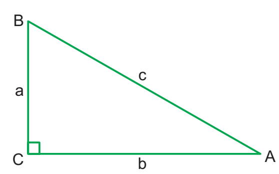
$a^2 + c^2 = b^2$

$E = \frac{\tan A + \cot A}{2\sec C\csc C} = \frac{\frac{a}{c} + \frac{c}{a}}{2.\frac{b}{a}.\frac{b}{c}} = \frac{\frac{a^2 + c^2}{ca}}{\frac{2b^2}{ca}}$

$E = \frac{b^2}{2b^2} = \frac{1}{2}$

Clave C

9.



Dato: $\dfrac{\tan A \cdot \cot B}{1 - \text{sen}A} = \dfrac{\tan B \cot A}{1 - \cos B}$

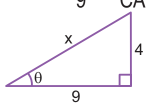
$\Rightarrow \dfrac{\frac{a}{b} \cdot \frac{a}{b}}{1 - \frac{a}{c}} = \dfrac{\frac{b}{a} \cdot \frac{b}{a}}{1 - \frac{a}{c}}$

$\dfrac{a^2}{b^2} = \dfrac{b^2}{a^2} \Rightarrow a = b$

$\therefore \tan A + \tan B = \dfrac{a}{b} + \dfrac{b}{a} = 2$

Clave B

10. $\tan\theta = \dfrac{4}{9} = \dfrac{CO}{CA}$



$x^2 = 4^2 + 9^2$

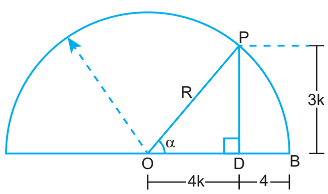
$x^2 = 97 \Rightarrow x = \sqrt{97}$

$\text{sen}\theta \cdot \cos\theta = \dfrac{CO}{H} \cdot \dfrac{CA}{H} = \dfrac{4 \cdot 9}{x^2} = \dfrac{36}{97}$

Clave D

Resolución de problemas

11. De los datos: $\alpha = m\angle POD \wedge \tan\alpha = \dfrac{3}{4}$



Luego:
PD = 3k, OD = 4k

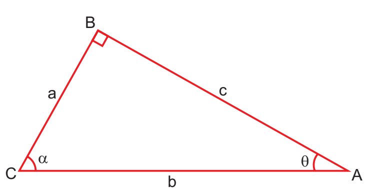
En el ⊿ PDO:
$R^2 = (4k)^2 + (3k)^2$
$R^2 = 16k^2 + 9k^2$
$R^2 = 25k^2$
$R = 5k$

Del gráfico:
$R = 4k + 4$
$5k = 4k + 4$
$k = 4$

Nos piden:
$R = 5k = 5(4)$
$\therefore R = 20$

Clave B

12. En el ⊿ ABC, sea α el mayor de los ángulos agudos y θ el menor:



Por dato:

$\text{sen}\alpha = \dfrac{c}{b} = \dfrac{21}{29}$

$\Rightarrow c = 21k; \ b = 29k$

Por el teorema de Pitágoras:
$a^2 + c^2 = b^2$
$a^2 + (21k)^2 = (29k)^2$
$a^2 = (29k)^2 - (21k)^2$
$a^2 = (29k + 21k)(29k - 21k)$
$a^2 = (50k)(8k)$
$a^2 = 400\,k^2$
$a = 20k$ ... (1)

De la figura:

Nos piden: $\tan\theta = \dfrac{a}{c}$

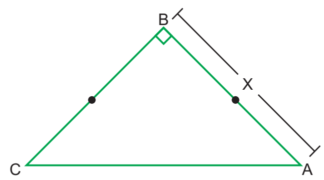
De (1):

$\tan\theta = \dfrac{a}{c} = \dfrac{20k}{21k}$

$\therefore \tan\theta = \dfrac{20}{21}$

Clave D

13. De los datos:



ABC: triángulo rectángulo isósceles
BC = AB = x ... (1)
Por el teorema de Pitágoras:
$BC^2 + AB^2 = AC^2$
$x^2 + x^2 = AC^2$
$2x^2 = AC^2$
$AC = x\sqrt{2}$ ... (2)

Por dato:
$AC + AB + BC = 20 + 10\sqrt{2}$

De (1) y (2):
$AC + AB + BC = x + x + x\sqrt{2}$

Entonces:
$2x + x\sqrt{2} = 20 + 10\sqrt{2}$
$x(2 + \sqrt{2}) = 10(2 + \sqrt{2})$
$x = 10$

Nos piden el lado de menor longitud.

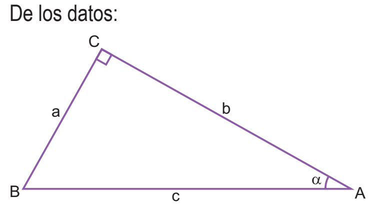
$\therefore AB = BC = 10$

Clave D

Nivel 2 (página 19) Unidad 1

Comunicación matemática

14. De los datos:

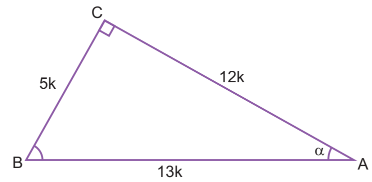


Donde:

$\dfrac{a}{b} = \dfrac{5}{12} \Rightarrow a = 5k; \quad b = 12k$

Por el teorema de Pitágoras:
$c^2 = a^2 + b^2$
$c^2 = (5k)^2 + (12k)^2$
$c^2 = 25k^2 + 144k^2$
$c^2 = 169k^2$
$c = 13k$

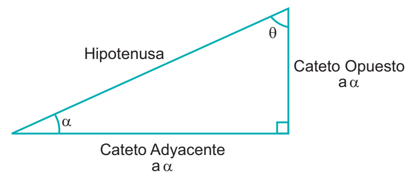
Finalmente, el triángulo rectángulo será:



$\therefore \csc A = \dfrac{13}{5}$

Clave C

15. Sea el ángulo α en un triángulo rectángulo:



I. Para α, su tangente es igual a la razón entre su cateto opuesto y su cateto adyacente, respectivamente.

(b)

II. De la figura, θ es el complemento de α, luego. El lado opuesto al ángulo θ, es cateto adyacente del ángulo α.

(a)

III. En la figura, el cateto adyacente al ángulo complementario de α (θ) es el cateto opuesto a α. Luego, la razón entre la hipotenusa y el cateto opuesto de α es igual a la cosecante de α.

(d)

IV. Para cualquier triángulo rectángulo, el lado opuesto al ángulo recto (90°) es la hipotenusa.

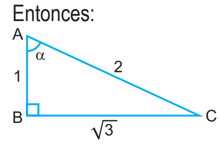
(c)

Clave D

Razonamiento y demostración

16. Dato: $\sec\alpha = 2$

Entonces:

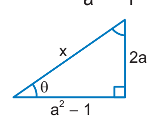


Piden:
$C = \sec\alpha + \csc\alpha$

$C = 2 + \dfrac{2}{\sqrt{3}} = \dfrac{6 + 2\sqrt{3}}{3}$

Clave B

17. $\tan\theta = \dfrac{2a}{a^2 - 1} = \dfrac{CO}{CA}$

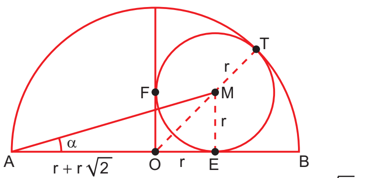


$x^2 = (2a)^2 + (a^2 - 1)^2$
$x^2 = 4a^2 + a^4 - 2a^2 + 1$
$x^2 = a^4 + 2a^2 + 1 = (a^2 + 1)^2$
$x = a^2 + 1$

$\therefore \text{sen}\theta = \dfrac{2a}{a^2 + 1}$

Clave A

18.

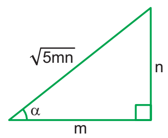


$\Rightarrow \cot\alpha = \dfrac{AE}{ME} = \dfrac{r+r\sqrt{2}+r}{r} = \dfrac{(2+\sqrt{2})r}{r}$

$\therefore \cot\alpha = 2+\sqrt{2}$

Clave A

19.



$m^2+n^2 = (\sqrt{5mn})^2$

$m^2+n^2 = 5mn$

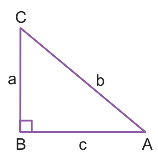
$E = \tan\alpha + \cot\alpha$

$E = \dfrac{n}{m} + \dfrac{m}{n} = \dfrac{n^2+m^2}{mn} = \dfrac{5mn}{mn}$

$\therefore\ E = 5$

Clave D

20.



$\text{sen}A = 4\tan C$

$\dfrac{a}{b} = \dfrac{4c}{a}$

$a^2 = 4bc$ ...(1)

$E = \sqrt[3]{\cot^2 C - 4\sec A + 7}$

$E = \sqrt[3]{\left(\dfrac{a}{c}\right)^2 - 4.\dfrac{b}{c} + 7}$

$E = \sqrt[3]{\dfrac{a^2}{c^2} - \dfrac{4b}{c} + 7}$

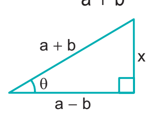
$E = \sqrt[3]{\dfrac{a^2-4bc}{c^2} + 7}$ ...(2)

Reemplazando (1) en (2):

$E = \sqrt[3]{0+7} = \sqrt[3]{7}$

Clave E

21. $\cos\theta = \dfrac{a-b}{a+b} = \dfrac{CA}{H}$



$(a+b)^2 = x^2 + (a-b)^2$

$a^2+2ab+b^2 = x^2+a^2-2ab+b^2$

$x^2 = 4ab$

$x = 2\sqrt{ab}$

Luego:

$E = (\sec\theta - \tan\theta)(\sqrt{a}+\sqrt{b})$

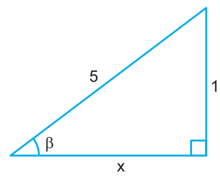
$E = \left(\dfrac{a+b}{a-b} - \dfrac{2\sqrt{ab}}{a-b}\right)(\sqrt{a}+\sqrt{b})$

$E = \dfrac{(\sqrt{a}-\sqrt{b})^2}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})}(\sqrt{a}+\sqrt{b})$

$E = \sqrt{a}-\sqrt{b}$

Clave A

22.



$\text{sen}\beta = \dfrac{1}{5} = \dfrac{CO}{H}$

Por el teorema de Pitágoras:

$1^2 + x^2 = 5^2$

$1 + x^2 = 25 \Rightarrow x^2 = 24$

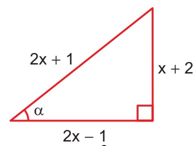
$x = 2\sqrt{6}$

$\Rightarrow M = \left(\dfrac{x}{1}\right)^2 + 5\sqrt{6}\left(\dfrac{x}{5}\right) = (2\sqrt{6})^2 + \sqrt{6}(2\sqrt{6})$

$\therefore\ M = 36$

Clave A

23.



$(x+2)^2 + (2x-1)^2 = (2x+1)^2$

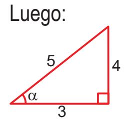
$(x+2)^2 + 4x^2 - 4x + 1 = 4x^2 + 4x + 1$

$x^2 + 4x + 4 - 4x = 4x$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

Luego: $\Rightarrow x = 2$



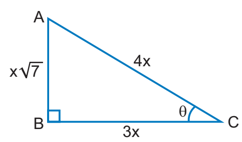
$E = \dfrac{\csc\alpha + \cot\alpha}{\sec\alpha - \tan\alpha}$

$\therefore E = \dfrac{\frac{5}{4}+\frac{3}{4}}{\frac{5}{3}-\frac{4}{3}} = \dfrac{2}{\frac{1}{3}} = 6$

Clave C

Resolución de problemas

24.



El menor ángulo agudo es C.

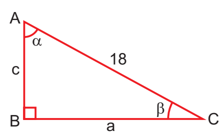
Piden:

$L = \text{sen}\theta\tan\theta$

$L = \dfrac{x\sqrt{7}}{4x} \cdot \dfrac{x\sqrt{7}}{3x} = \dfrac{7}{12}$

Clave D

25.



Dato:

$\cos\alpha + \cos\beta = \dfrac{1}{3}$

$\dfrac{c}{18} + \dfrac{a}{18} = \dfrac{1}{3}$

$c + a = \dfrac{18}{3}$

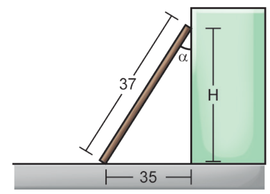
$c + a = 6$

Piden la suma de los catetos, entonces:

$c + a = 6$

Clave E

26. Del enunciado:



Por el teorema de Pitágoras:

$H^2 + 35^2 = 37^2$

$H^2 = 37^2 - 35^2$

$H^2 = (37-35)(37+35)$

$H^2 = (2)(72)$

$H^2 = 144$

$H = 12$

Nos piden:

$\cos\alpha = \dfrac{H}{37}$

$\therefore\ \cos\alpha = \dfrac{12}{37}$

Clave B

Nivel 3 (página 20) Unidad 1

Comunicación matemática

27.

A) $\text{sen}\alpha = \dfrac{2}{9} < 1$ ... Correcto

B) $\csc\theta = \sqrt{7} - \sqrt{5} < 1$ ... Incorrecto

C) $\text{sen}\omega = \sqrt{2} - 1 < 1$ ... Correcto

D) $\sec\omega = \sqrt{11} - \sqrt{5} > 1$ ... Correcto

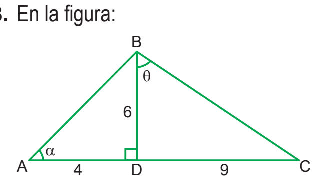
E) $\cos\theta = \dfrac{1}{\sqrt{3}+\sqrt{2}} \cdot \dfrac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$

$\cos\theta = \dfrac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2}$

$\cos\theta = \sqrt{3}-\sqrt{2} < 1$ ... Correcto

Clave B

28. En la figura:



Por el teorema de Pitágoras:

$AB^2 = 4^2 + 6^2$

$AB^2 = 16 + 36$

$AB^2 = 52$

$AB = 2\sqrt{13}$

$BC^2 = 6^2 + 9^2$

$BC^2 = 36 + 81$

$BC^2 = 117$

$BC = 3\sqrt{13}$

A) $\text{sen}\alpha = \dfrac{BD}{AB} = \dfrac{6}{2\sqrt{13}} = \dfrac{3}{\sqrt{13}} = \dfrac{3\sqrt{13}}{13}$

$\therefore\ \text{sen}\alpha = \dfrac{3\sqrt{13}}{13}$ ... Correcto

B) $\sec\theta = \dfrac{BC}{BD} = \dfrac{3\sqrt{13}}{6} = \dfrac{\sqrt{13}}{2}$

$\therefore\ \sec\theta = \dfrac{\sqrt{13}}{2}$ ... Incorrecto

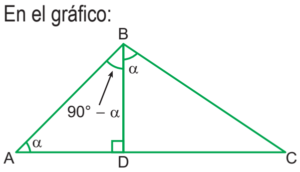
C) De la figura:

$\tan\alpha = \dfrac{6}{4} = \dfrac{3}{2};\quad \tan\theta = \dfrac{9}{6} = \dfrac{3}{2}$

$\Rightarrow\ \tan\alpha = \tan\theta$

$\Rightarrow\ \alpha = \theta$

En el gráfico:



$m\angle ABD + m\angle DBC = 90°$

$\therefore$ ABC triángulo rectángulo recto en B.

... Correcto

D) De la parte C; $\alpha = \theta$

... Correcto

E) Del gráfico:

$\cos\alpha = \dfrac{AD}{AB} = \dfrac{4}{2\sqrt{13}} = \dfrac{2}{\sqrt{13}}$

$\therefore$ AB es a AD como $\sqrt{13}$ es a 2.

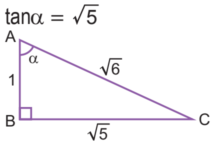
... Correcto

Clave B

Razonamiento y demostración

29. Del dato:

$\tan\alpha = \sqrt{5}$

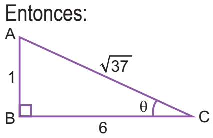


También:

$\tan\theta = \cos^2\alpha$

$\tan\theta = \left(\dfrac{1}{\sqrt{6}}\right)^2 = \dfrac{1}{6}$

Entonces:



Piden:

$L = 37\text{sen}^2\theta + 6\text{sen}^2\alpha$

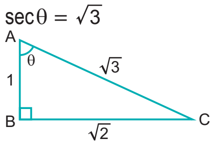
$L = 37\left(\dfrac{1}{\sqrt{37}}\right)^2 + 6\left(\dfrac{\sqrt{5}}{\sqrt{6}}\right)^2$

$\therefore \quad L = 1 + 5 = 6$

Clave C

30. Dato:

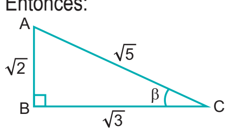
$\sec\theta = \sqrt{3}$



También:

$\tan\beta = \text{sen}\theta = \dfrac{\sqrt{2}}{\sqrt{3}}$

Entonces:



Piden:

$L = \tan^2\theta + 5\cos^2\beta$

$L = \left(\dfrac{\sqrt{2}}{1}\right)^2 + 5\left(\dfrac{\sqrt{3}}{\sqrt{5}}\right)^2$

$L = 2 + 3 = 5$

Clave C

31. Propiedad: $A + C = 90°$

$\Rightarrow \text{sen}A = \cos C$

$\dfrac{2x+1}{6x+1} = \dfrac{3x-1}{7x-1}$

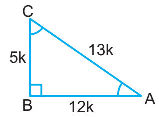
$14x^2 + 5x - 1 = 18x^2 - 3x - 1$

$\Rightarrow 0 = 4x^2 - 8x$

$0 = 4x(x-2)$

$\Rightarrow x = 2$

$\text{sen}A = \dfrac{2(2)+1}{6(2)+1} = \dfrac{5}{13}$



$12k = 6$

$k = \dfrac{1}{2}$

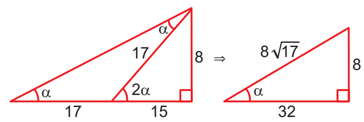
Perímetro = 30k

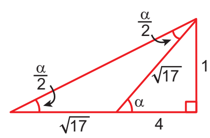
$\therefore$ Perímetro $= 30\left(\dfrac{1}{2}\right) = 15$

Clave D

32. $0 < \alpha < 45°$

$\cot 2\alpha = \dfrac{15}{8} = \dfrac{CA}{AO} \quad \Rightarrow \quad H = 17$





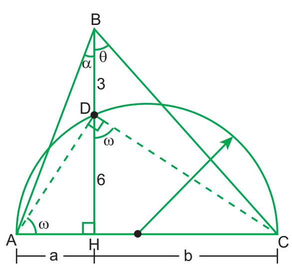
$E = (\sqrt{17} - 4)\cot\dfrac{\alpha}{2}$

$E = (\sqrt{17} - 4)(\sqrt{17} + 4) = (\sqrt{17})^2 - 4^2$

$\therefore E = 1$

Clave A

33.



ADC: triángulo rectángulo

$m\angle DAH = m\angle CDH = \omega$

Nos piden:

$\tan\alpha \,.\, \tan\theta = \dfrac{a}{9} \cdot \dfrac{b}{9} = \dfrac{ab}{81} \quad \text{... (1)}$

En el ⊿ ADC:

$\tan\omega = \dfrac{6}{a} = \dfrac{b}{6} \Rightarrow ab = 36 \quad \text{... (2)}$

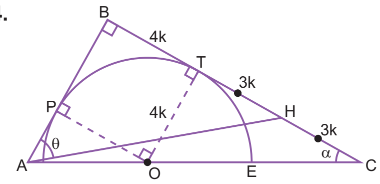
De (2) en (1):

$\tan\alpha \,.\, \tan\theta = \dfrac{ab}{81} = \dfrac{36}{81}$

$\therefore \tan\alpha \,.\, \tan\theta = \dfrac{4}{9}$

Clave E

34.



Datos:

3BT = 4TH

$\dfrac{BT}{TH} = \dfrac{4}{3} \Rightarrow BT = 4k,\ TH = 3k$

Luego, en el cuadrado PBTO, tenemos:

BT = OT = 4k

Sea $\alpha = m\angle BCA$; en los triángulos rectángulos CTO y CBA:

$\tan\alpha = \dfrac{4k}{6k} = \dfrac{AB}{10k} \Rightarrow AB = \dfrac{20k}{3}$

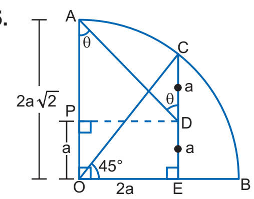
Finalmente en el ⊿ ABH:

$\tan\theta = \dfrac{7k}{AB} = \dfrac{7k}{\frac{20}{3}k}$

$\therefore \tan\theta = \dfrac{21}{20}$

Clave A

35.



Dato:

$\overset{\frown}{AC} = \overset{\frown}{CB} \Rightarrow m\angle COB = 45°$

Entonces:

⊿ CEO: isósceles

Sea:

CD = DE = a

$\Rightarrow$ CE = OE = 2a

Por el teorema de Pitágoras:

$OC^2 = OE^2 + CE^2$

$OC^2 = (2a)^2 + (2a)^2$

$OC^2 = 2(2a)^2$

$OC = 2a\sqrt{2}$

Donde:

OC: radio del sector circular AOB.

$\Rightarrow AO = OC = 2a\sqrt{2}$

Se traza $\overline{DP} \perp \overline{AO}$, luego en el rectángulo OPDE tenemos:

OP = DE = a

En el ⊿ APD:

$m\angle DAP = \theta$

$AP = AO - PO = 2a\sqrt{2} - a$

PD = OE = 2a

Finalmente:

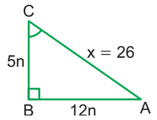
$\cot\theta = \dfrac{AP}{PD} = \dfrac{2a\sqrt{2} - a}{2a}$

$\therefore \cot\theta = \dfrac{2\sqrt{2} - 1}{2}$

Clave C

Resolución de problemas

36.



$\tan A = \dfrac{5}{12}$

$x^2 = (5n)^2 + (12n)^2$

$x^2 = 169n^2$

$x = 13n = 26$

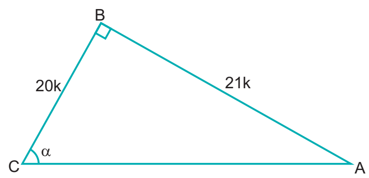
$\Rightarrow n = 2$

$\text{Área} = \dfrac{5n \,.\, 12n}{2} = 30n^2 = 30(2)^2$

$\therefore \text{Área} = 120 \text{ m}^2$

Clave B

37. Sea el triángulo ABC rectángulo, y α el ángulo cuya tangente es igual a 1,05.



$\tan\alpha = \dfrac{AB}{BC} = 1{,}05 = \dfrac{105}{100} = \dfrac{21}{20}$

$\Rightarrow AB = 21k,\ BC = 20k$

Por el teorema de Pitágoras:

$AC^2 = (20k^2) + (21k^2)$

$AC^2 = 841k^2$

$AC = 29k$

Dato:

Perímetro: $2p = 140$ u ... (1)

Luego:

$2p = 20k + 21k + 29k = 70k$

$2p = 70k$

De (1):

$70k = 140$ u

$k = 2$ u

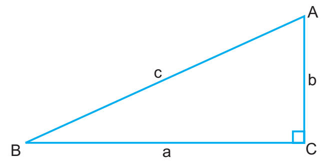
Nos piden el lado mayor (hipotenusa):

$AC = 29k = 29(2)$

$\therefore AC = 58$ u

Clave B

38. Sea el triángulo rectángulo ABC (recto en C):



Datos:

$a + b = 6$ ... (1)

$\text{sen}A \,.\, \text{sen}B = 0{,}22$ ... (2)

De (2):

$\text{sen}A \,.\, \text{sen}B = \dfrac{a}{c} . \dfrac{b}{c} = \dfrac{ab}{c^2} = 0{,}22$

$\Rightarrow ab = (0{,}22)c^2$ ... (3)

De (1):

Elevamos la expresión al cuadrado

$(a + b)^2 = 6^2$

$a^2 + b^2 + 2ab = 36$ ... (4)

Por teorema de Pitágoras

$a^2 + b^2 = c^2$

En (4):

$c^2 + 2ab = 36$ ... (5)

(3) en (5):

$c^2 + 2(0{,}22)c^2 = 36$

$1{,}44c^2 = 36$

$\dfrac{144}{100}c^2 = 36$

$c^2 = \dfrac{36.100}{144}$

$c = \dfrac{6.10}{12}$

$\therefore c = 5$ m

Clave C

APLICAMOS LO APRENDIDO (página 22) Unidad 1

1. $\cos(7x-3°)\sec(5x+7°)=1$

$$\Rightarrow (7x-3°)=(5x+7°)$$
$$7x-5x=7°+3°$$
$$2x=10°$$
$$\therefore\ x=5°$$

Clave E

2. $\tan 7x=\cot 3x$

$$\Rightarrow 7x+3x=90°$$
$$10x=90°$$
$$\therefore\ x=9°$$

Clave C

3. $\tan(\alpha+\beta)=\cot 70°$

$$\Rightarrow (\alpha+\beta)+70°=90°$$
$$\alpha+\beta=20° \quad ...(I)$$
$$\text{sen}(\alpha-\beta)=\cos 84°$$
$$\Rightarrow (\alpha-\beta)+84°=90°$$
$$\alpha-\beta=6° \quad ...(II)$$

De (I) y (II):

$$\alpha=13° \quad \wedge \quad \beta=7°$$

Clave C

4. $E=\dfrac{\text{sen}10°+\tan 20°+\sec 30°}{\csc 60°+\cot 70°+\cos 80°}$

Se cumple:

$\text{sen}10°=\cos 80°$

$\tan 20°=\cot 70°$

$\csc 60°=\sec 30°$

Reemplazando en el denominador, tenemos:

$$E=\frac{\text{sen}10°+\tan 20°+\sec 30°}{\sec 30°+\tan 20°+\text{sen}10°}=1$$

$\therefore\ E=1$

Clave B

5. $\tan x\underbrace{\tan 50°\tan 40°}_{\cot 50°}\tan 30°=1$

$$\tan x\underbrace{\tan 50°\cot 50°}_{1}\tan 30°=1$$
$$\tan x\underbrace{\tan 30°}=1$$
$$\tan x\underbrace{\cot 60°}_{\tan 60°}=1$$
$$\Rightarrow \tan x=\tan 60°$$
$$\therefore\ x=60°$$

Clave E

6. $\text{sen}3\alpha=\cos 75°$

$$\Rightarrow 3\alpha+75°=90°$$
$$3\alpha=15°$$
$$\Rightarrow \alpha=5°$$
$$\tan 2\beta=\cot 80°$$
$$\Rightarrow 2\beta+80°=90°$$
$$2\beta=10°$$
$$\Rightarrow \beta=5°$$
$$\sec(\alpha+\beta)=\csc\theta$$
$$\sec(5°+5°)=\csc\theta$$
$$\sec 10°=\csc\theta$$
$$\Rightarrow 10°+\theta=90°$$
$$\theta=80°.\left(\frac{\pi\ \text{rad}}{180°}\right)$$
$$\therefore\ \theta=\frac{4\pi}{9}\ \text{rad}$$

Clave D

7. $\text{sen}\alpha-\cos 2\beta=0$

$$\text{sen}\alpha=\cos 2\beta$$
$$\Rightarrow \alpha+2\beta=90° \quad ...(I)$$
$$\cos\alpha\sec(3\beta-10°)=1$$
$$\Rightarrow \alpha=3\beta-10° \quad ...(II)$$

Reemplazando (II) en (I):

$$(3\beta-10°)+2\beta=90°$$
$$5\beta=100°$$
$$\beta=20°$$
$$\Rightarrow \alpha=50°$$

Piden: $\alpha-\beta$

$$\alpha-\beta=50°-20°=30°$$
$$\therefore\ \alpha-\beta=30°$$

Clave C

8. $\dfrac{\text{sen}(2x+25°)\cos 56°}{\cos(x+5°)\text{sen}34°}=\sqrt{(\sqrt{3})^2-2}$

$$\frac{\text{sen}(2x+25°)\text{sen}34°}{\cos(x+5°)\text{sen}34°}=\sqrt{3-2}=1$$
$$\text{sen}(2x+25°)=\cos(x+5°)$$
$$\Rightarrow (2x+25°)+(x+5°)=90°$$
$$3x=60°$$
$$\Rightarrow x=20°$$

Piden:

$$E=[\cos(2x+10°)-\text{sen}2x+2]\frac{\sqrt{3}}{2}$$
$$E=[\cos 50°-\text{sen}40°+2]\frac{\sqrt{3}}{2}$$
$$E=[\text{sen}40°-\text{sen}40°+2]\frac{\sqrt{3}}{2}=2.\frac{\sqrt{3}}{2}=\sqrt{3}$$
$$\therefore\ E=\sqrt{3}$$

Clave E

9. $E=\dfrac{4\text{sen}x}{\cos(90°-x)}+\dfrac{2\text{sen}10°}{\cos 80°}+\dfrac{\tan 72°}{\cot 18°}$

$$E=\frac{4\text{sen}x}{(\text{sen}x)}+\frac{2\text{sen}10°}{(\text{sen}10°)}+\frac{\tan 72°}{(\tan 72°)}$$
$$E=4+2+1=7$$
$$\therefore\ E=7$$

Clave D

10. $\text{sen}(2x+y)\csc(2y+30°)=1$

$$\Rightarrow 2x+y=2y+30°$$
$$2x-y=30° \quad ...(I)$$
$$\tan(x+30°)=\cot(y+30°)$$
$$\Rightarrow (x+30°)+(y+30°)=90°$$
$$x+y=30° \quad ...(II)$$

De (I) y (II):

$x=20° \quad \wedge \quad y=10°$

Piden: $3x-2y$

$$3x-2y=3(20°)-2(10°)=60°-20°=40°$$
$$\therefore\ 3x-2y=40°$$

Clave D

11. De la expresión:

$$\tan\left(\frac{3\pi}{2}-5x\right)=\cot\left(x-\frac{\pi}{9}\right)$$

Ángulos complementarios:

$$\frac{3\pi}{2}-5x+x-\frac{\pi}{9}=\frac{\pi}{2}$$
$$\pi-\frac{\pi}{9}=4x$$
$$4x=\frac{8\pi}{9}$$
$$\therefore\ x=\frac{2\pi}{9}\ \text{rad}$$

Clave B

12. De la expresión:

$$\text{sen}(5x-1)°\underbrace{\sec 61°}_{\csc 29°}\ \underbrace{\csc 73°}_{\sec 17°}\cos 17°=1$$
$$\text{sen}(5x-1)°\csc 29°\ \underbrace{\sec 17°\cos 17°}_{1}=1$$
$$\text{sen}(5x-1)°\csc 29°=1$$
$$\Rightarrow (5x-1)°=29°$$
$$5x-1=29$$
$$5x=30$$
$$\therefore\ x=6$$

Clave E

13. De la expresión:

$$\csc(n+45)°=\sec(m-15)°$$

De ángulos complementarios:

$$(n+45)°+(m-15)°=90°$$
$$n+45+m-15=90$$
$$n+m=60$$
$$\therefore\ \frac{n+m}{2}=30$$

Clave A

14. De la expresión:

$$\sec(41-a)°\ .\ \cos(37+b)°=1$$

Razones trigonométricas recíprocas:

$$(41-a)°=(37+b)°$$
$$41-a=37+b$$
$$a+b=4$$
$$\therefore\ (a+b)^2=16$$

Clave B

PRACTIQUEMOS

Nivel 1 (página 24) Unidad 1

Comunicación Matemática

1. I. Para 2 ángulos θ y β complementarios se cumple:

$$\text{sen}\theta=\cos\beta$$

... (Falsa)

II. Del enunciado:

$$\tan(90°-\alpha)=\cot\alpha$$

$(90°-\alpha)$ y α son complementarios.

... (Verdadera)

III. α y θ son complementarios, se cumple:

$$\sec\alpha=\csc\theta$$

... (Verdadera)

Clave C

2. Del triángulo ABC:

$$\tan\alpha=\frac{6}{4}=\frac{3}{2}$$
$$\cot\theta=\frac{6}{9}=\frac{2}{3}$$

Luego:

$\tan\alpha \, . \, \cot\theta = \frac{3}{2} . \frac{2}{3} = 1$

$\tan\alpha \, . \, \cot\theta = 1$

Por razones trigonométricas recíprocas :
$\alpha = \theta$

Clave B

Razonamiento y demostración

3. $\tan 3x \tan(2x + 20°) = 1$
$\tan 3x \cot(70° - 2x) = 1$
Se debe cumplir:
$3x = 70° - 2x$
$5x = 70°$
$\Rightarrow x = 14°$

Clave B

4. $\text{sen}4x \, . \, \csc(x + 30°) = 1$
Se debe cumplir:
$4x = x + 30°$
$3x = 30°$
$\Rightarrow x = 10°$

Clave B

5. $\cos(3x - 10°) \, . \, \sec(x + 20°) = 1$

Se debe cumplir:
$3x - 10° = x + 20°$
$2x = 30°$
$\Rightarrow x = 15°$

Clave C

6. $\tan 2x \, . \cot(60° - x) = 1$

Se debe cumplir:
$2x = 60° - x$
$3x = 60°$
$\Rightarrow x = 20°$

Clave A

7. $\text{sen}\,a = \cos b$

$\Rightarrow a + b = 90°$
$\Rightarrow \text{sen}\,b = \cos a$
$\therefore W = \frac{\text{sen}\,b}{\cos a} = 1$

Clave A

8. $\text{sen}2x \, . \, \csc(3x - 1°) = 1$
$2x = 3x - 1°$
$\Rightarrow x = 1°$

Clave A

9. $\text{sen}4x \, . \, \csc(x + 30°) = 1$
$4x = x + 30°$
$3x = 30° \quad \Rightarrow \quad x = 10°$

Clave B

10. $\cos(3x - 10°) \, . \, \sec(x + 20°) = 1$
$3x - 10° = x + 20° \quad \Rightarrow \quad 2x = 30°$
$\Rightarrow \quad x = 15°$

Clave C

11. $\tan 5x \, . \, \cot(x + 20°) = 1$
$5x = x + 20°$
$4x = 20° \Rightarrow x = 5°$

Clave A

Resolución de problemas

12. De los datos:

$\frac{\alpha}{2} + \frac{\beta}{3} = 15°$

$\frac{3\alpha + 2\beta}{6} = 15°$

3α y 2β son complementarios, luego:
$\text{sen}3\alpha = \cos 2\beta$
$\frac{\text{sen}3\alpha}{\cos 2\beta} = 1$

$\therefore \frac{2\text{sen}3\alpha}{\cos 2\beta} = 2$

Clave B

13. Del enunciado:

$\text{sen}3a = \cos 2a$

Por razones de ángulos complementarios:
$3a + 2a = 90°$
$5a = 90°$
$a = 18°$

Reemplazamos en E:
$E = \frac{(\text{sen}18° + \cos 72°) . \csc 18°}{2}$

72° y 18° complementarios, luego:

$E = \text{sen}18° \, . \, \csc 18°$
$\therefore E = 1$

Clave B

Nivel 2 (página 24) Unidad 1

Comunicación Matemática

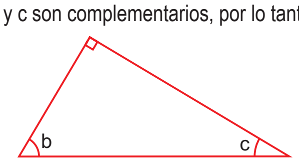
14. I. De los datos:
$\sec a \, . \, \cos b = 1$
$\Rightarrow \; a = b$ ... (1)
Además:
$\sec c = \csc b$
$\Rightarrow \; c + b = 90°$ ... (2)
(1) en (2):
$c + a = 90°$
$\therefore$ a y c son complementarios.

(Correcto)

II. De (2):
c y b son complementarios

(Incorrecto)

III. b y c son complementarios, por lo tanto:

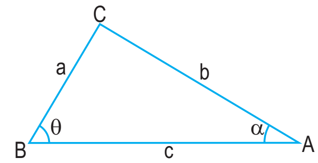


Existe un triángulo rectángulo de ángulos agudos b y c.

(Correcto)

Clave E

15. Del triángulo ABC:



Cumple con el teorema de Pitágoras:
$$a^2 + b^2 = c^2$$
Luego:
$m\angle C = 90° \quad \wedge \quad \theta + \alpha = 90°$

I. De la expresión:
$\tan\theta = \cot\left(\frac{\pi}{2} - \alpha\right)$

$\Rightarrow \; \theta + \frac{\pi}{2} - \alpha = \frac{\pi}{2}$
$\theta = \alpha$

Pero:
$a \neq b \; \Rightarrow \; \theta \neq \alpha$
Por contradicción

... (Falso)

II. De la expresión:
$\tan\theta \, . \, \underbrace{\tan\alpha}_{\cot\left(\frac{\pi}{2} - \alpha\right)} = 1$

$\tan\theta \, . \, \cot\left(\frac{\pi}{2} - \alpha\right) = 1$

$\Rightarrow \quad \theta = \frac{\pi}{2} - \alpha$

$\theta + \alpha = \frac{\pi}{2}$

... (Verdadero)

III. α y θ son complementarios.
Luego:
$\sec\theta = \csc\alpha$
$\frac{\csc\alpha}{\sec\theta} = 1 \quad \Rightarrow \quad \frac{5\csc\alpha}{2\sec\theta} = \frac{5}{2}$

... (Falso)

Clave C

Razonamiento y demostración

16. $E = (3\text{sen}36° + 4\cos 54°)\csc 36°$

$E = \underbrace{3\text{sen}36°\csc 36°}_{1} + 4\cos 54°\underbrace{\csc 36°}_{\sec 54°}$

$E = 3 + 4\underbrace{\cos 54°\sec 54°}_{1}$

$E = 3 + 4 = 7$

Clave D

17. $C = \frac{\text{sen}10°}{\cos 80°} + \frac{\tan 20°}{\cot 70°}$

$C = \frac{\cos 80°}{\cos 80°} + \frac{\cot 70°}{\cot 70°}$

$C = 1 + 1 \quad \Rightarrow \quad C = 2$

Clave B

18. $\tan(2x - 16°)\tan(x + 40°) = 1$
$\tan(2x - 16°)\cot(50° - x) = 1$

Se debe cumplir:
$2x - 16° = 50° - x$
$3x = 66°$
$\Rightarrow x = 22°$

Clave B

19. $E = (2\text{sen}10° + 3\cos 80°)\csc 10°$

$E = \underbrace{2\text{sen}10°\csc 10°}_{1} + 3\underbrace{\cos 80°}_{\text{sen}10°}\csc 10°$

$E = 2 + 3\underbrace{\text{sen}10°\csc 10°}_{1}$

$E = 2 + 3 = 5$

Clave C

20. $\tan\alpha = \cot 2\alpha$

$\alpha + 2\alpha = 90°$
$3\alpha = 90°$
$\alpha = 30°$

Piden: $\dfrac{\text{sen}\alpha + \cos 2\alpha}{\text{sen}\alpha}$

$$\frac{\text{sen}30° + \cos 60°}{\text{sen}30°} = \frac{\frac{1}{2}+\frac{1}{2}}{\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

Clave B

21. $\tan(b + 15°) \,.\, \cot(2b - 5°) = 1$

$b + 15° = 2b - 5°$
$\Rightarrow b = 20°$

Clave A

22. $\cos(x + 5°) \,.\, \csc(3x + 5°) = 1$

$\cos(x + 5°) = \text{sen}(3x + 5°)$
$x + 5° + 3x + 5° = 90°$
$4x = 80°$
$\Rightarrow x = 20°$

Clave D

23. De la expresión:

$$\text{sen}\left(\frac{n+m}{2} - 17°\right) = \cos\left(\frac{n-m}{2} + 63°\right)$$

Por razones trigonométricas de ángulos complementarios:

$$\frac{n+m}{2} - 17° + \frac{n-m}{2} + 63° = 90°$$

$$\frac{n+m+n-m}{2} + 46° = 90°$$

$$\frac{2n}{2} = 44°$$

$$\therefore\ n = 44°$$

Clave D

Resolución de problemas

24. Del enunciado:

$$\text{sen}3\alpha = \cos\left(\frac{\alpha}{2} + 20°\right)$$

3α y $\left(\frac{\alpha}{2} + 20°\right)$ agudos complementarios, luego:

$$3\alpha + \frac{\alpha}{2} + 20° = 90°$$

$$\frac{7\alpha}{2} = 70°$$

$$\alpha = 20°$$

Luego:

$\alpha = 20° . \dfrac{\pi \text{ rad}}{180°}$

$\therefore\ \alpha = \dfrac{\pi}{9}$ rad

Clave D

25. Del enunciado, sean α y β los ángulos mencionados:
$\text{sen}\alpha \,.\, \text{sen}\beta = \cos\alpha \,.\, \cos 70°$ ... (1)

α y β complementarios:
$\text{sen}\beta = \cos\alpha$

En (1):
$\text{sen}\alpha \,.\, \cos\alpha = \cos\alpha \,.\, \cos 70°$
$\text{sen}\alpha = \cos 70°$

70° y α agudos y complementarios:
$\alpha + 70° = 90°$
$\alpha = 20°$
$\Rightarrow \beta = 70° \quad \wedge \quad \alpha = 20°$

Se pide:

$S = 2\alpha + \dfrac{\beta}{2}$

$S = 2 \,.\, 20° + \dfrac{70°}{2}$

$\therefore\ S = 75°$

Clave B

Nivel 3 (página 25) Unidad 1

Comunicación Matemática

26. Para ángulos complementarios α y β:
$RT(\alpha) = \text{co-}RT(\beta)$
$\Rightarrow \alpha + \beta = 90°$

A) De la igualdad:
$\tan(a - b) = \cot(2b + c)$
$\Rightarrow$ $(a - b)$ y $(2b + c)$ complementarios:
$a - b + 2b + c = a + b + c = 90°$

(Correcto)

B) Análogamente:
$\text{sen}(3b - 5c + 2a) = \cos(6c - a - 2b)$
$(3b - 5c + 2a) + (6c - a - 2b) = a + b + c = 90°$
Son complementarios.

(Correcto)

C) $\tan\left(90° + \dfrac{3c}{5} + \dfrac{b}{2} - \dfrac{a}{3}\right) = \cot\left(\dfrac{4a}{3} + \dfrac{b}{2} + \dfrac{2c}{5}\right)$

Luego:

$$\left(90° + \frac{3c}{5} + \frac{b}{2} - \frac{a}{3}\right) + \left(\frac{4a}{3} + \frac{b}{2} + \frac{2c}{5}\right) = 90° + a + b + c$$

$$\left(90° + \frac{3c}{5} + \frac{b}{2} - \frac{a}{3}\right) + \left(\frac{4a}{3} + \frac{b}{2} + \frac{2c}{5}\right) = 180°$$

No son complementarios.

(Incorrecto)

D) $\csc\left(\dfrac{b+c}{2} + \dfrac{a}{3}\right) = \sec\left(\dfrac{4a+3b+3c}{6}\right)$

$$\left(\frac{b+c}{2} + \frac{a}{3}\right) + \left(\frac{4a+3b+3c}{6}\right) = a + b + c$$

$$\left(\frac{b+c}{2} + \frac{a}{3}\right) + \left(\frac{4a+3b+3c}{6}\right) = 90°$$

Son complementarios. (Correcto)

Clave C

27. De la expresión:
$\text{sen}\theta \,.\, \sec\alpha \,.\, \tan(37° + 2p) \,.\, \tan(p - 13°) = 1$
θ y α complementarios, entonces:
$\cos\alpha \,.\, \sec\alpha \,.\, \tan(37° + 2p) \,.\, \tan(p - 13°) = 1$
$\tan(37° + 2p) \,.\, \tan(p - 13°) = 1$ ... (1)

Luego:
$\tan(p - 13°) = \cot(90° - (p - 13°))$
$\tan(p - 13°) = \cot(103° - p)$

En (1):
$\tan(37° + 2p) \,.\, \cot(103° - p) = 1$

tan y cot razones recíprocas:
$37° + 2p = 103° - p$
$3p = 66°$
$p = 22°$

I. $p = 22° . \dfrac{\pi \text{ rad}}{180°}$

$p = \dfrac{11\pi}{90}$ rad

(V)

II. De $(4p + 1°)$ y $5p$:
$(4p + 1°) + 5p = 9p + 1°$
$(4p + 1°) + 5p = 9(22°) + 1°$
$(4p + 1°) + 5p = 199°$
$\therefore$ $(4p + 1°)$ y $5p$ no son complementarios.

(F)

III. De la expresión (reemplazamos $p = 22°$):
$\tan(2p - 15°) \,.\, \cot(95° - 3p) = \tan 29° \,.\, \cot 29°$
$\tan(2p - 15°) \,.\, \cot(95° - 3p) = 1$
$\therefore$ Son recíprocos.

(V)

Clave A

Razonamiento y demostración

28. $E = [\cos 20° \,.\, \sec 20° + \tan 58° \,.\, \cot 58°]^{\text{sen}10° \,.\, \csc 10°}$

Sabemos:
$\cos 20° \,.\, \sec 20° = 1$
$\tan 58° \,.\, \cot 58° = 1$
$\text{sen}10° \,.\, \csc 10° = 1$
Reemplazamos:
$E = [1 + 1]^1 = 2^1 = 2$

Clave D

29. $\cos(\alpha + 10°) = \dfrac{1}{\csc(\alpha + 10°)}$ ...(1)

Sabemos:
$\text{sen}\beta \,.\, \csc\beta = 1 \Rightarrow \dfrac{1}{\csc\beta} = \text{sen}\beta$

$\Rightarrow \dfrac{1}{\csc(\alpha + 10°)} = \text{sen}(\alpha + 10°)$

Reemplazamos en la expresión (1):
$\cos(\alpha + 10°) = \text{sen}(\alpha + 10°)$
$\Rightarrow (\alpha + 10°) + (\alpha + 10°) = 90°$
$2\alpha + 20° = 90°$
$2\alpha = 70°$
$\therefore \alpha = 35°$

Clave A

30. $\text{sen}(x + 60°) = \cos(y - 37°)$

$\Rightarrow (x + 60°) + (y - 37°) = 90°$

$x + y = 67°$ ...(I)

$\tan(45° + x) = \cot(z - 37°)$

$\Rightarrow (45° + x) + (z - 37°) = 90°$

$x + z = 82°$ ...(II)

$\sec(z + 30°) = \csc(y - 15°)$

$(z + 30°) + (y - 15°) = 90°$

$z + y = 75°$ ...(III)

Sumando las expresiones (I), (II) y (III) tenemos:

$2(x + y + z) = 67° + 82° + 75°$

$2(x + y + z) = 224°$

$\therefore\ x + y + z = 112°$

Clave A

31. De la condición:

$\text{sen}(2a + b) = \cos(a + 2b)$

Se debe cumplir:

$2a + b + a + 2b = 90°$

$\Rightarrow 3a + 3b = 90°$

Piden:

$P = \dfrac{\text{sen}3a}{\cos 3b} + \dfrac{\text{sen}3b}{\cos 3a}$

$P = \dfrac{\text{sen}(90° - 3b)}{\cos 3b} + \dfrac{\text{sen}(90° - 3a)}{\cos 3a}$

$P = \dfrac{\cos 3b}{\cos 3b} + \dfrac{\cos 3a}{\cos 3a}$

$P = 1 + 1 = 2$

Clave B

32. $\text{sen}2x = \cos 40°$

$2x + 40° = 90°$

$\Rightarrow x = 25°$

$\tan 3x \cot y = 1$

$\tan 75° \cot y = 1$

$\Rightarrow y = 75°$

Piden: $y - x = 75° - 25° = 50°$

Clave B

33. $\text{sen}2x \,.\, \csc(48° - x) = 1$

Se debe cumplir:

$2x = 48° - x$

$\Rightarrow x = 16°$

$\tan 4x \,.\, \cot 8y = 1$

Se debe cumplir:

$4x = 8y \Rightarrow 4 \times 16° = 8y$

$\Rightarrow y = 8°$

Piden:

$\dfrac{x}{y} = \dfrac{16°}{8°} = 2$

Clave B

34. Del dato:

$\text{sen}2x = \cos 5x \Rightarrow 2x + 5x = 90°$

$7x = 90°$

Piden:

$E = \underbrace{\tan 3x \tan 4x}_{\cot 4x} + \underbrace{\text{sen}x \sec 6x}_{\cos 6x}$

$E = \cot 4x \tan 4x + \cos 6x \sec 6x$

$E = 1 + 1 = 2$

Clave D

35. $\text{sen}(a + 30°) = \cos(4a + 10°)$

$\Rightarrow a + 30° + 4a + 10° = 90°$

$5a = 50°$

$\Rightarrow a = 10°$

$\tan(b + 20°) \,.\, \cot 50° = 1$

$\tan(b + 20°) = \tan(50°)$

$\Rightarrow b + 20° = 50°$

$\Rightarrow b = 30°$

$\therefore a + b = 40°$

Clave E

Resolución de problemas

36. Del enunciado se deduce:

$\text{sen}\alpha \,.\, \csc\alpha \,.\, \cos\alpha \,.\, \sec\alpha \,.\, \tan\alpha = 1$

$\tan\alpha = 1$

$\Rightarrow \alpha = 45°$

Clave D

37. Del triángulo:

$a + b + c = 180°$

$\dfrac{a + b + c}{2} = 90°$ ... (1)

En m:

$m = \dfrac{\tan\left(\frac{a+b}{2}\right)}{\cot\frac{c}{2}} \,.\, \dfrac{\sec\left(\frac{c+b}{2}\right)}{\csc\frac{a}{2}} \,.\, \dfrac{\text{sen}\left(\frac{a+c}{2}\right)}{\cos\frac{b}{2}}$

Luego de (1):

$\left(\dfrac{a+b}{2}\right)$ y $\dfrac{c}{2}$ son complementarios.

$\left(\dfrac{c+b}{2}\right)$ y $\dfrac{a}{2}$ son complementarios.

$\left(\dfrac{a+c}{2}\right)$ y $\dfrac{b}{2}$ son complementarios.

En m:

$m = (1) \,.\, (1) \,.\, (1)$

$m = 1$

$\therefore\ \dfrac{3m}{2} = \dfrac{3}{2}$

Clave B

MARATÓN MATEMÁTICA (página 27)

1. Sea el ángulo: $\dfrac{S}{C} = \dfrac{9}{10} = k \Rightarrow S = 9k$

$C = 10k$

Reemplazamos igualdades:

$\dfrac{9k}{3} - 12 = x + 3 \Rightarrow 3k = x + 15$ ... (I)

$\dfrac{10k}{2} + 6 = x + 31 \Rightarrow 5k = x + 25$ ... (II)

(II) – (I):

$2k = 10 \Rightarrow k = 5\,;\ S = 9k$

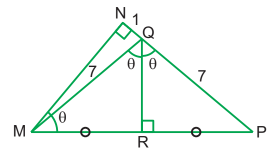
$S = 45°$

Transformamos a radianes:

$45 \times \dfrac{\pi \text{ rad}}{180} = \dfrac{\pi}{4}$ rad

Clave C

2. Del gráfico:



$MN^2 + 1^2 = 7^2 \Rightarrow MN^2 = 48$

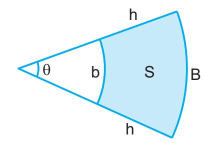
Nos piden:

$\tan^2\theta = \left(\dfrac{NP}{MN}\right)^2 = \dfrac{8^2}{MN^2} = \dfrac{64}{48}$

$\tan^2\theta = \dfrac{4}{3}$

Clave D

3. El área de un trapecio circular está definido por:



$S = \dfrac{h}{2}(b + B)$

Entonces tenemos:

$S = \dfrac{h}{2}(b + B)$

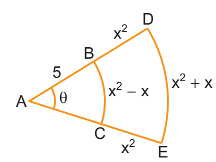
$12M^2 = \dfrac{3M}{2}(2M + k)$

$8M = 2M + k$

$\therefore\ k = 6M$

Clave A

4.



Del gráfico:

$5\theta = x^2 - x$

$(5 + x^2)\theta = x^2 + x$

$\Rightarrow \dfrac{5}{5 + x^2} = \dfrac{x^2 - x}{x^2 + x}$

$\dfrac{5}{5 + x^2} = \dfrac{x - 1}{x + 1}$

$5x + 5 = 5x - 5 + x^3 - x^2$

$10 = x^3 - x^2$

En P:

$P = x^2 - x^3 + 15$

$P = -10 + 15$

$\therefore P = 5$

Clave E

5. 1 vuelta $\Rightarrow$ 360°

$\Rightarrow$ 2 vueltas $\Rightarrow$ 720°

$x" = \dfrac{720°}{1000} = \dfrac{720°}{1000} \times \dfrac{60'}{1°} \times \dfrac{60"}{1'} = 2592"$

$\therefore x" = 2592"$

Clave B

6. Del gráfico:

$y^\circ = (x+3)^\circ - (3-x)^g$

$y^\circ = (x+3)^\circ + \frac{9}{10}(x-3)^\circ$

$y = \frac{(10x + 30 + 9x - 27)^\circ}{10} \Rightarrow 10y = 19x + 3$

Nos piden:

$P = 19x - 10y \Rightarrow P = 19x - 19x - 3$

$\therefore\ P = -3$

Clave E

7. Perímetro $= m\overset{\frown}{AB} + AC + CE + ED + BD$

$2p = (0{,}41\pi)2 + (2) + \sqrt{5^2 - 4^2} + \sqrt{(5^2 - 4^2)} + 2$

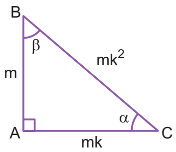
$2p = 0{,}82\pi + 2 + 3 + 3 + 2$

$2p = 10 + 0{,}82\pi = 12{,}57$

$\therefore\ 2p = 12{,}57$ m

Clave A

8.



Por Pitágoras:

$m^2 + (mk)^2 = (mk^2)^2$

$m^2 + m^2k^2 = m^2k^4$

$1 + k^2 = k^4$

$k^2 = \frac{\sqrt{5}+1}{2}$

$\Rightarrow\ k = \sqrt{\frac{\sqrt{5}+1}{2}}$

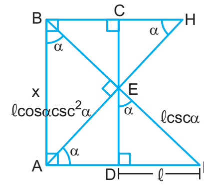
Nos piden:

$\tan\beta = \frac{mk}{m} = k$

$\therefore\ \tan\beta = \sqrt{\frac{\sqrt{5}+1}{2}}$

Clave A

9.



$\frac{AE}{AB} = \operatorname{sen}\alpha$

$\frac{\ell\cos\alpha\csc^2\alpha}{\operatorname{sen}\alpha} = x$

$\therefore\ x = \ell\cos\alpha\csc^3\alpha$

Clave B

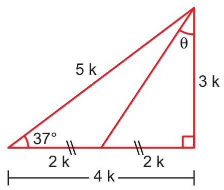
APLICAMOS LO APRENDIDO (página 30) Unidad 2

1. En el gráfico, ⊿ notable de 37° y 53°.

Luego:

$\tan\theta = \dfrac{2k}{3k}$

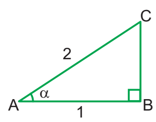
$\therefore\ \tan\theta = \dfrac{2}{3}$



Clave B

2. $\cos\alpha = \dfrac{\cot 45°}{2} = \dfrac{1}{2}$

$\cos\alpha = \dfrac{1}{2}$, α es agudo.



Luego:
ABC ⊿ notable 30° y 60°.
$\Rightarrow \alpha = 60°$
$\tan\alpha = \tan 60°$
$\therefore\ \tan\alpha = \sqrt{3}$

Clave D

3. $\operatorname{sen}4x\csc(x + 60°) = 1$

Se debe cumplir: $4x = x + 60° \Rightarrow x = 20°$

Piden: $\tan(2x + 5°) = \tan(2(20) + 5°)$
$= \tan 45°$

$\therefore\ \tan(2x + 5°) = 1$

Clave A

4. $\tan 2x \cot 40° = 1$

Se debe cumplir:
$2x = 40° \Rightarrow x = 20°$

Piden: $\operatorname{sen}3x = \operatorname{sen}3(20°)$

$= \operatorname{sen}60° = \dfrac{\sqrt{3}}{2}$

Clave B

5. ADB ⊿ notable de 30° y 60°:

$\operatorname{sen}30° = \dfrac{AD}{AB} = \dfrac{1}{2}$

$AB = 2AD = 2 . 3 \Rightarrow AB = 6$

ABC ⊿ notable de $\dfrac{37°}{2}$ y $\dfrac{143°}{2}$:

$\tan\dfrac{37°}{2} = \dfrac{BC}{AB} = \dfrac{1}{3}$

$BC = \dfrac{AB}{3} = \dfrac{6}{3}$

$\therefore\ BC = 2$

Clave A

6. $M = \sqrt{2}\csc 8° + \sqrt{3}\tan 60° + \sqrt{10}\csc\dfrac{37°}{2}$

$M = \sqrt{2} . 5\sqrt{2} + \sqrt{3} . \sqrt{3} . + \sqrt{10} . \sqrt{10}$

$M = 10 + 3 + 10$

$\therefore\ M = 23$

Clave D

7. $\sec 2x = \csc x$

sec y csc son co-razones, luego x y 2x son complementarios:

$2x + x = 90°$
$3x = 90°$
$x = 30°$

Piden:
$L = \csc x + \sec 2x$
$L = \csc 30° + \sec 60°$
$L = 2 + 2$
$\therefore\ L = 4$

Clave C

8. $\operatorname{sen}3x = \cos 2x$

Se debe cumplir:
$3x + 2x = 90°$
$5x = 90°$
$\Rightarrow x = 18°$

Piden:

$Q = \operatorname{sen}^2\dfrac{5x}{3}\tan(3x - 1°)$

$Q = \operatorname{sen}^2\dfrac{5 . 18°}{3}\tan(3(18°) - 1°)$

$Q = \operatorname{sen}^2 30° \tan 53°$

$Q = \left(\dfrac{1}{2}\right)^2 \dfrac{4}{3}$

$\therefore Q = \dfrac{1}{3}$

Clave C

9. $\tan 3x \cot(x + 40°) = 1$

Se debe cumplir:
$3x = x + 40° \Rightarrow x = 20°$

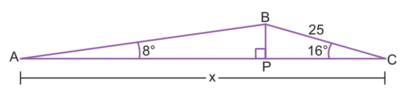
Piden: $\operatorname{sen}3x = \operatorname{sen}3(20°)$

$= \operatorname{sen}60°$

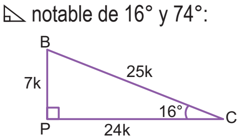
$= \dfrac{\sqrt{3}}{2}$

Clave C

10.

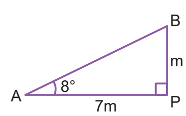


- BPC ⊿ notable de 16° y 74°:



$BC = 25k = 25 \Rightarrow\ k = 1$
$PC = 24 . 1 = 24$
$BP = 7 . 1 = 7$

- BPA ⊿ notable de 8° y 82°:



$BP = m = 7 \Rightarrow\ m = 7$
$AP = 7m = 7 . 7$
$AP = 49$

Luego:
$x = AP + PC = 49 + 24$
$\therefore\ x = 73$

Clave E

11. $P = \cos\dfrac{143°}{2} . \sqrt{10} + \operatorname{sen}\dfrac{127°}{2} . \sqrt{20} + \sec 82° . \sqrt{2}$

$P = \dfrac{1}{\sqrt{10}} . \sqrt{10} + \dfrac{2}{\sqrt{5}} . \sqrt{20} + 5\sqrt{2} . \sqrt{2}$

$P = 1 + 4 + 10$

$\therefore\ P = 15$

Clave E

12. $\cos 2x = \dfrac{P - 1}{P + 1}$

Para $x = 30°$

$\cos 60° = \dfrac{P - 1}{P + 1}$

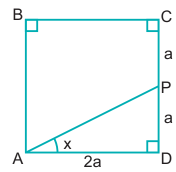
$\dfrac{1}{2} = \dfrac{P - 1}{P + 1}$

$P + 1 = 2P - 2$

$\therefore\ P = 3$

Clave C

13.



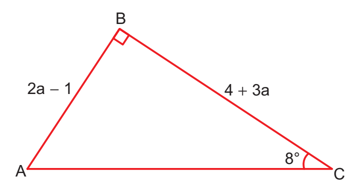
ADP ⊿ notable de $\dfrac{53°}{2}$ y $\dfrac{127°}{2}$

$x = \dfrac{53°}{2}$

$\therefore\ \tan 2x = \tan 53° = \dfrac{4}{3}$

Clave E

14. ABC ⊿ notable de 8° y 82°:



$\tan 8° = \dfrac{1}{7} = \dfrac{2a - 1}{4 + 3a}$

$\Rightarrow \dfrac{2a - 1}{4 + 3a} = \dfrac{1}{7}$

$14a - 7 = 4 + 3a$
$11a = 11$
$a = 1$
$\therefore\ 2a = 2$

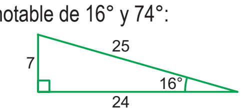
Clave A

PRACTIQUEMOS

Nivel 1 (página 32) Unidad 2

Comunicación matemática

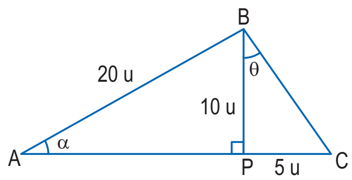
1. Del ⊿ notable de 16° y 74°:



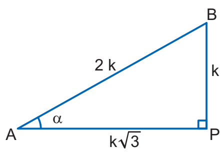
$\therefore\ \cot 16° = \dfrac{24}{7}$

Clave C

2.



I. Se observa:



APB ◺ notable de 30° y 60°:

$\alpha = 30°$

$2\alpha = 60°$

...(Falsa)

II. De lo anterior:

$BP = k$

$k = 10$

$AP = k\sqrt{3}$

$AP = 10\sqrt{3}$

$\Rightarrow AC = AP + PC$

$AC = 10\sqrt{3} + 5$

...(Falsa)

III. En el triángulo BPC:

BPC ◺ notable de $\frac{53°}{2}$ y $\frac{127°}{2}$:

$\theta = \frac{53°}{2}$

$2\theta = 53°$

$\Rightarrow 90° - 2\theta = 90° - 53°$

$90° - 2\theta = 37°$

El complemento de 2θ es igual a 37°.

... (Verdadera)

Clave B

Razonamiento y demostración

3. $P = \tan 45° + \sqrt{3}\tan 30° + \tan^2 60°$

$\therefore P = 1 + \sqrt{3}\left(\frac{1}{\sqrt{3}}\right) + \left(\sqrt{3}\right)^2 = 1 + 1 + 3 = 5$

Clave A

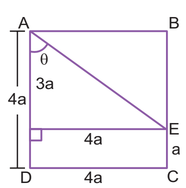
4. $E(x) = \text{sen}^2 2x + \tan^2 3x - \sec 4x$

$E(15°) = \text{sen}^2 30° + \tan^2 45° - \sec 60°$

$\therefore E(15°) = \left(\frac{1}{2}\right)^2 + 1 - 2 = \frac{1}{4} - 1 = \frac{-3}{4}$

Clave D

5.



$\Rightarrow \tan\theta = \frac{4}{3}$

Clave D

6. $W = \tan 45° + \text{sen}60° \,.\, \cos 30° + \text{sen}^2 45°$

$W = 1 + \frac{\sqrt{3}}{2} \,.\, \frac{\sqrt{3}}{2} + \left(\frac{\sqrt{2}}{2}\right)^2$

$\therefore W = 1 + \frac{3}{4} + \frac{1}{2} = 1 + \frac{5}{4} = \frac{9}{4}$

Clave C

7. $M = \tan 2x \,.\, \sec 3x \,.\, \text{sen}4x$

Como: $x = 15°$

Entonces: $M = \tan 30° \,.\, \sec 45° \,.\, \text{sen}60°$

$\therefore M = \frac{1}{\sqrt{3}} \,.\, \sqrt{2} \,.\, \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{2}$

Clave E

8. $E = \sec x \tan 2x - 2\cot\left(\frac{3x}{2}\right)$

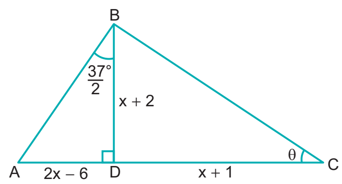
Reemplazando: $x = 30°$

$E = \sec 30° \tan 60° - 2\cot(45°)$

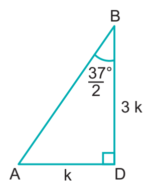
$\therefore E = \frac{2}{\sqrt{3}} \,.\, \sqrt{3} - 2 \,.\, 1 = 2 - 2 = 0$

Clave C

9.



ADB ◺ notable: $\frac{37°}{2}$ y $\frac{143°}{2}$:



$\frac{AD}{BD} = \frac{k}{3k}$

$\frac{2x - 6}{x + 2} = \frac{1}{3}$

Luego:

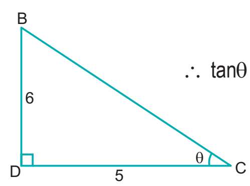
$3(2x - 6) = x + 2$

$6x - 18 = x + 2$

$5x = 20$

$x = 4$

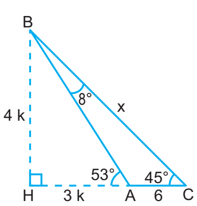
En el ◺ BDC:



$\therefore \tan\theta = \frac{6}{5}$

Clave D

10.



Prologamos $\overline{CA}$ y trazamos $\overline{BH}$($BH = CH$).

BHA ◺ notable de 53° y 37°:

$BH = 4k \;\wedge\; AH = 3k$

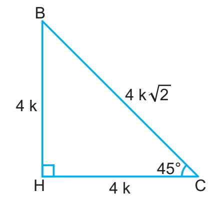
BHC ◺ notable de 45°:

$BH = HC$

$4k = 3k + 6$

$k = 6$

Además:



$BC = 4k\sqrt{2}$

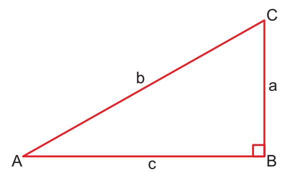
$x = 4k\sqrt{2} = 4 \,.\, 6\sqrt{2}$

$\therefore x = 24\sqrt{2}$

Clave C

Resolución de problemas

11. Sea el triángulo rectángulo ABC:



Datos:

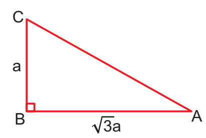
$3\tan A = \tan C$

$3\frac{a}{c} = \frac{c}{a}$

$3a^2 = c^2$

$\sqrt{3}\,a = c$

Luego:



ABC ◺ notable de 30° y 60°:

$m\angle A = 30°$

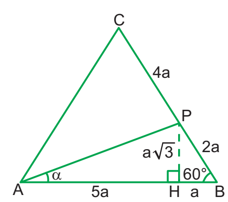
$m\angle C = 60°$

Finalmente, el menor ángulo agudo es: 30°

$\therefore \cos 30° = \frac{\sqrt{3}}{2}$

Clave E

12. Sea el triángulo:



Trazamos $\overline{PH} \perp \overline{AB}$, luego:

PHB ◺ 60° y 30°:

$BP = 2a$, $HB = a$, $PH = a\sqrt{3}$

Por dato: $2BP = PC$

$2(2a) = PC$

$PC = 4a \Rightarrow BC = 6a$

$\triangle ABC$ equilátero:

$AB = BC$

$AH + HB = 6a$

$AH + a = 6a$

$AH = 5a$

$\tan\alpha = \frac{PH}{AH} = \frac{a\sqrt{3}}{5a}$

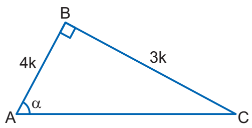
$\therefore \tan\alpha = \frac{\sqrt{3}}{5}$

Clave B

Nivel 2 (página 32) Unidad 2

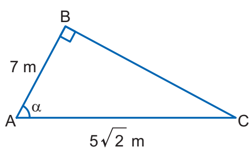
Comunicación matemática

13. I. De la proporción: $\frac{a}{c} = \frac{3}{4}$



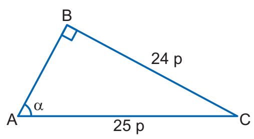
⊿ notable de 37° y 53° $\Rightarrow \alpha = 37° \Rightarrow$ Ic

II. Análogamente para $\frac{b}{c} = \frac{5\sqrt{2}}{7}$



⊿ notable de 8° y 82° $\Rightarrow \alpha = 8° \Rightarrow$ IIa

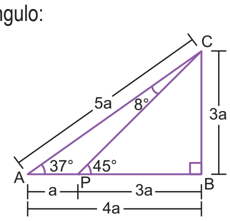
III. Finalmente para $\frac{a}{b} = \frac{24}{25}$



⊿ notable de 74° y 16° $\Rightarrow \alpha = 74° \Rightarrow$ IIIb

Clave C

14. ABC ⊿ notable de 37° y 53°.
PBC ⊿ notable de 45°.
En el triángulo:



I. Se observa:

$\frac{AC}{PB} = \frac{5a}{3a} = \frac{5}{3}$

$\therefore$ La razón de AC y PB es $\frac{5}{3}$.

... Correcta

II. Se tiene que:

$\frac{BC}{AP} = \frac{3a}{a} \Rightarrow BC = 3AP$

$\therefore$ BC es el triple de AP.

... Correcta

III. Finalmente:

$\frac{AB}{AP} = \frac{4a}{a} \Rightarrow \frac{AB}{4} = AP$

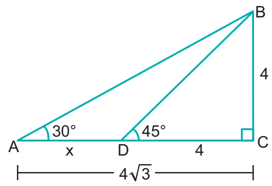
$\therefore$ AP es la cuarta parte de AB.

... Incorrecta

Clave E

Razonamiento y demostración

15.



$\Rightarrow x = 4\sqrt{3} - 4 \Rightarrow x = 4(\sqrt{3} - 1)$

Clave C

16. $2x\,\text{sen}30° + \cos^2 60° = \sqrt{3}\tan 60° + 2x\tan 45°$

$2x \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 = \sqrt{3} \cdot \sqrt{3} + 2 \cdot x \cdot 1$

$x + \frac{1}{4} = 3 + 2x$

$\therefore x = -\frac{11}{4}$

Clave E

17. $E = (\sec 60° + \tan 45°)\sec 53° + \sqrt{6}\tan 60° \sec 45°$

$E = (2 + 1) \cdot \frac{5}{3} + \sqrt{6} \cdot \sqrt{3} \cdot \sqrt{2}$

$E = 3 \cdot \frac{5}{3} + \sqrt{36}$

$\therefore E = 5 + 6 = 11$

Clave B

18. $M = \sqrt{\dfrac{\text{sen}^2 30° + \sec^3 60° - \cos^4 45°}{\tan 37° \cdot \tan 53° \cdot \cot 45° \cdot \csc^6 45°}}$

$M = \sqrt{\dfrac{\left(\frac{1}{2}\right)^2 + (2)^3 - \left(\frac{1}{\sqrt{2}}\right)^4}{\frac{3}{4} \cdot \frac{4}{3} \cdot 1 \cdot \left(\frac{\sqrt{2}}{1}\right)^6}}$

$\therefore M = \sqrt{\dfrac{\frac{1}{4} + 8 - \frac{1}{4}}{8}} = \sqrt{\frac{8}{8}} = 1$

Clave A

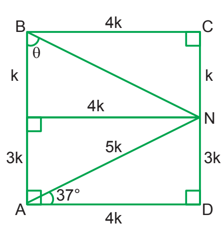
19. $P = \sqrt{\dfrac{\sqrt{3}\cos^2 60° \cdot \sec 30° \cdot \tan 45°}{\sec^2 45° - 6\cos 30° + \tan^3 60°}}$

$P = \sqrt{\dfrac{\sqrt{3} \cdot \left(\frac{1}{2}\right)^2 \cdot \frac{2}{\sqrt{3}} \cdot 1}{(\sqrt{2})^2 - 6 \cdot \frac{\sqrt{3}}{2} + (\sqrt{3})^3}} = \sqrt{\dfrac{\frac{1}{2}}{2}}$

$\therefore P = \sqrt{\frac{1}{4}} = \frac{1}{2}$

Clave C

20.

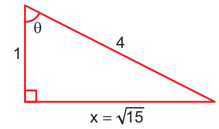


$\therefore \tan\theta = \frac{4k}{k} = 4$

Clave C

21. $\cos\theta = \cos^2 60° = \left(\frac{1}{2}\right)^2$

$\cos\theta = \frac{1}{4}$; (θ agudo)



Teorema de Pitágoras:

$1 + x^2 = 4^2$

$x^2 = 15$

$x = \sqrt{15}$

Finalmente:

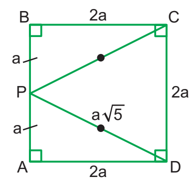
$C = \sec\theta + \tan^2\theta \Rightarrow C = 4 + (\sqrt{15})^2$

$\therefore C = 19$

Clave E

Resolución de problemas

22. Del enunciado:



CDP △ isósceles: PC = PD

Luego: ⊿CBP ≅ ⊿DAP

BP = PA

P: punto medio de $\overline{AB}$

PAD ⊿ notable de $\frac{53°}{2}$ y $\frac{127°}{2}$:

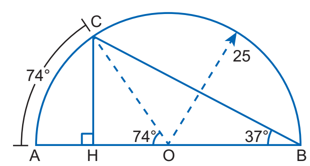
$PD = a\sqrt{5}$

Dato: $AD = 6 \Rightarrow 2a = 6 \Rightarrow a = 3$

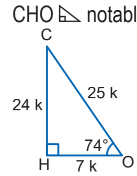
$\therefore PD = 3\sqrt{5}$

Clave B

23. Sea $\overset{\frown}{AB}$ semicircunferencia de centro O:



CHO ⊿ notable de 16° y 74°:



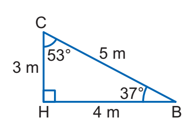
$CO = 25k$

$25 = 25k$

$\Rightarrow k = 1$

$\Rightarrow CH = 24 \wedge HO = 7$

CHB ⊿ notable de 37° y 53°:



$CH = 3\,m$

$24 = 3\,m \Rightarrow m = 8$

$BH = 4\,m = 4 \cdot 8$

$\therefore BH = 32$

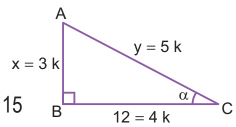
Clave E

Nivel 3 (página 33) Unidad 2

Comunicación matemática

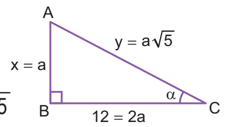
24. I. Si $\alpha = 37°$

$BC = 4k$
$12 = 4k$
$\Rightarrow k = 3$
$\Rightarrow x = 9 \wedge y = 15$



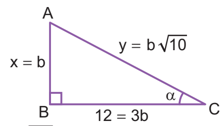
II. Si $\alpha = 53°/2$

$BC = 2a$
$12 = 2a$
$\Rightarrow a = 6$
$\Rightarrow x = 6 \wedge y = 6\sqrt{5}$



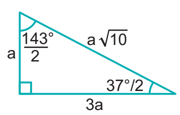
III. Si $\alpha = 37°/2$

$BC = 3b$
$12 = 3b$
$\Rightarrow b = 4$
$\Rightarrow x = 4 \wedge y = 4\sqrt{10}$



Clave D

25. Del ⊿ notable de $\frac{37°}{2}$ y $\frac{143°}{2}$:



$$\text{sen}\,\frac{143°}{2} = \frac{3a}{a\sqrt{10}} = \frac{3}{\sqrt{10}}$$

$$\therefore\ \text{sen}\,\frac{143°}{2} = \frac{3\sqrt{10}}{10}$$

Clave C

Razonamiento y demostración

26. $37x\tan^2 30° - 5x\sec^2 30° = 7\tan 45° + 5\sec 60°$

$$37x\left(\frac{1}{\sqrt{3}}\right)^2 - 5x\left(\frac{2}{\sqrt{3}}\right)^2 = 7(1) + 5(2)$$

$$\frac{37x}{3} - \frac{20x}{3} = 7 + 10$$

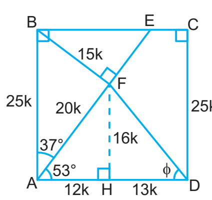
$$\frac{17x}{3} = 17$$

$$x = 3$$

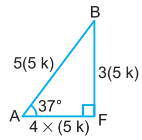
En P:
$P = \tan^2 15x + \cot^2 10x$
$P = \tan^2(15 \,.\, 3) + \cot^2(10 \,.\, 3)$
$P = \tan^2 45° + \cot^2 30°$
$P = 1^2 + (\sqrt{3})^2 \qquad \therefore\ P = 4$

Clave C

27.

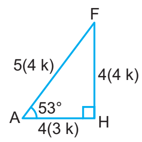


Trazamos $\overline{FH} \perp \overline{AD}$, sea $AF = 20k$.
AFB ⊿ notable de 37° y 53°:



$\overline{AB} = 5(5k)$
$\overline{AB} = 25k$

AHF ⊿ notable de 53° y 37°:



$FH = 16k$
$AH = 12k$

Finalmente:

$$\tan\phi = \frac{FH}{HD} = \frac{16\,k}{25\,k - 12\,k} = \frac{16\,k}{13\,k}$$

$$\therefore\ \tan\phi = \frac{16}{13}$$

Clave C

28. $M = \sqrt{6}\,\text{sen}30°\cos 45°\tan 60°$

$$M = \sqrt{6}\,.\,\frac{1}{2}\,.\,\frac{1}{\sqrt{2}}\,.\,\sqrt{3} = \frac{3\sqrt{2}}{2\sqrt{2}} = \frac{3}{2}$$

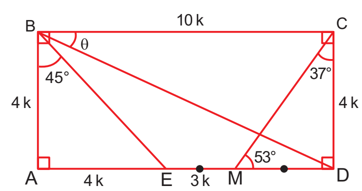
$N = \tan 30°\ \tan 45°\tan 60°$

$$N = \frac{1}{\sqrt{3}}\,.\,1\,.\,\sqrt{3} = 1$$

$$\therefore\ M + N = \frac{3}{2} + 1 = \frac{5}{2}$$

Clave B

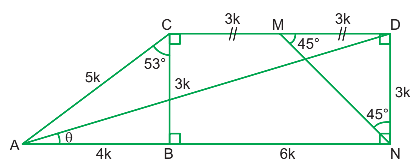
29.



$$\therefore\ \tan\theta = \frac{4k}{10k} = \frac{4}{10} = 0,4$$

Clave B

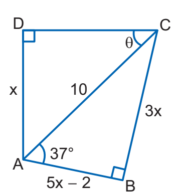
30.



$$\Rightarrow \tan\theta = \frac{3k}{10k} \qquad \therefore\ \tan\theta = 0,3$$

Clave B

31.



$$\tan 37° = \frac{3}{4}$$

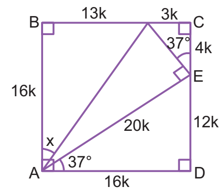
$$\Rightarrow \frac{3x}{5x - 2} = \frac{3}{4}$$

$$\Rightarrow x = 2$$

$(AC)^2 = (AB)^2 + (BC)^2$
$(AC)^2 = 8^2 + 6^2 = 100$
$\Rightarrow AC = 10 \qquad \therefore\ \text{sen}\theta = \frac{2}{10} = 0,2$

Clave B

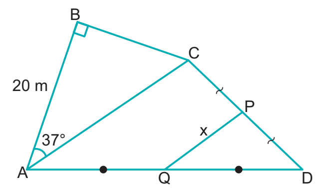
32. Si: $AE = 20k$



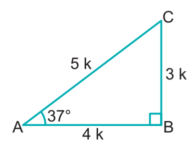
$$\therefore\ \tan x = \frac{13k}{16k} = \frac{13}{16}$$

Clave B

33. Del enunciado:



En △ACD:
P y Q son puntos medios de $\overline{CD}$ y $\overline{AD}$
$AC = 2PQ$
$AC = 2x$
ABC ⊿ notable de 37° y 53°:

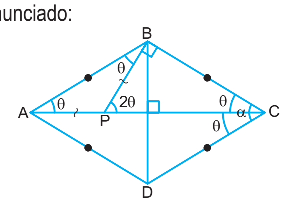


$AB = 20$ m
$4k = 20 \Rightarrow k = 5$
$AC = 5k = 5 \,.\, 5$
$\Rightarrow AC = 25$ m

Finalmente:
$AC = 2x$
$25 = 2x$
$\therefore\ x = 12,5$ m

Clave D

34. Del enunciado:



△APB isósceles:
$m\angle PAB = m\angle PBA = \theta$

En el ⊿ PBC:
$2\theta + \theta = 90°$
$3\theta = 90°$
$\theta = 30°$

Luego:
$\alpha = 2\theta$
$\alpha = 2 \,.\, 30°$
$\alpha = 60°$

$$M = 5\text{sen}^2(60° - 7°) + \text{sen}^2\left(\frac{2\,.\,60° + 23°}{2}\right)$$

$$M = 5\text{sen}^2 53° + \text{sen}^2\frac{143°}{2}$$

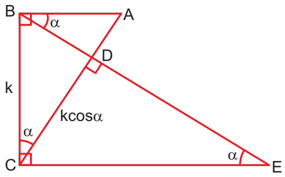
$$M = 5\left(\frac{4}{5}\right)^2 + \left(\frac{3}{\sqrt{10}}\right)^2 \Rightarrow M = \frac{16}{5} + \frac{9}{10}$$

$\therefore\ M = 4,1$

Clave B

APLICAMOS LO APRENDIDO (página 35) Unidad 2

1.



Del gráfico: $\dfrac{DE}{DC} = \cot\alpha$

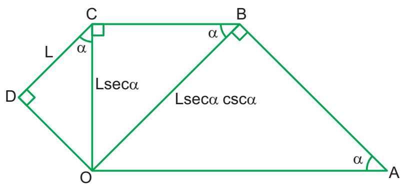
$DE = (DC)\cot\alpha$

$DE = (k\cos\alpha)\cot\alpha$

$\therefore DE = k\cos\alpha\cot\alpha$

Clave A

2.



Del gráfico: $\dfrac{BA}{BO} = \cot\alpha$

$BA = (BO)\cot\alpha$

$BA = L\sec\alpha\csc\alpha\cot\alpha$

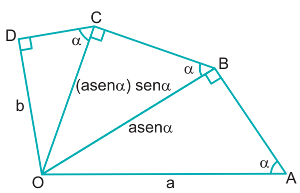
$BA = L\dfrac{1}{\cos\alpha}\cdot\dfrac{1}{\text{sen}\alpha}\cdot\dfrac{\cos\alpha}{\text{sen}\alpha}$

$BA = L\,.\,\dfrac{1}{\text{sen}^2\alpha} = L\,.\,\csc^2\alpha$

$\therefore BA = L\csc^2\alpha$

Clave B

3.



En el triángulo rectángulo ODC:

$\text{sen}\alpha = \dfrac{OD}{OC}$

$\text{sen}\alpha = \dfrac{b}{(a\text{sen}\alpha)\text{sen}\alpha}$

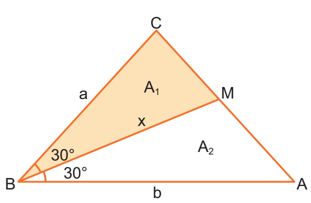
$\text{sen}\alpha = \dfrac{b}{a\text{sen}^2\alpha}$

$\text{sen}^3\alpha = \dfrac{b}{a}$

$\therefore \text{sen}\alpha = \sqrt[3]{\dfrac{b}{a}}$

Clave C

4.



Del gráfico:

$A_1 + A_2 = A_{total}$

$\dfrac{ax}{2}\text{sen}30° + \dfrac{xb}{2}\text{sen}30° = \dfrac{ab}{2}\text{sen}60°$

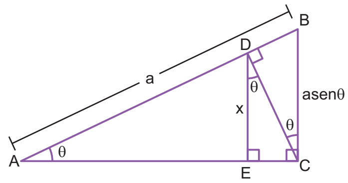
$x\text{sen}30°(a + b) = ab\text{sen}60°$

$x\left(\dfrac{1}{2}\right)(a + b) = ab\left(\dfrac{\sqrt{3}}{2}\right)$

$\therefore x = \dfrac{\sqrt{3}\,ab}{a + b}$

Clave C

5.



En el ◺BDC:

$\dfrac{DC}{BC} = \cos\theta$

$DC = (BC)\cos\theta$

$DC = (a\text{sen}\theta)\cos\theta \Rightarrow DC = a\text{sen}\theta\cos\theta$

En el ◺DEC:

$\dfrac{x}{DC} = \cos\theta$

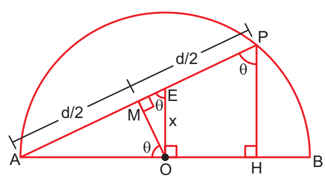
$x = (DC)\cos\theta$

$x = (a\text{sen}\theta\cos\theta)\cos\theta$

$\therefore x = a\text{sen}\theta\cos^2\theta$

Clave B

6. Se traza $\overline{OM} \perp \overline{AP}$.



En el ◺AMO: $OM = \dfrac{d}{2}\cot\theta$

En el ◺EMO:

$\dfrac{x}{OM} = \csc\theta$

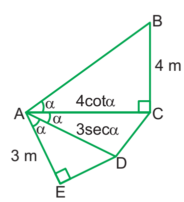
$x = (OM)\csc\theta$

$x = (\dfrac{d}{2}\cot\theta)\csc\theta$

$\therefore x = \dfrac{d}{2}\cot\theta\csc\theta$

Clave E

7.



Piden el área del ◺CAD: S

$S = \dfrac{(AC)(AD)}{2}\text{sen}\alpha$

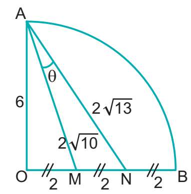
$S = \dfrac{(4\cot\alpha)(3\sec\alpha)}{2}.\text{sen}\alpha$

$S = 6\dfrac{\cos\alpha}{\text{sen}\alpha}.\dfrac{1}{\cos\alpha}.\text{sen}\alpha$

$\therefore S = 6\ m^2$

Clave A

8.



Por el teorema de Pitágoras:

$AM = 2\sqrt{10}$ (◺AOM)

$AN = 2\sqrt{13}$ (◺AON)

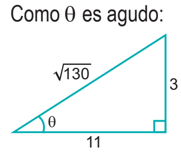
El área del △AMN, será:

$\dfrac{(\text{base})(\text{altura})}{2} = \dfrac{(AM)(AN)}{2}\text{sen}\theta$

$\dfrac{(2)(6)}{2} = \dfrac{(2\sqrt{10})(2\sqrt{13})}{2}\text{sen}\theta$

$\Rightarrow \text{sen}\theta = \dfrac{3}{\sqrt{130}}$

Como θ es agudo:

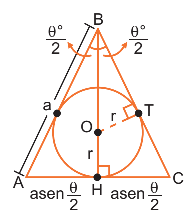


Piden: tanθ

$\therefore \tan\theta = \dfrac{3}{11}$

Clave A

9.



En el ◺BHA: $BH = a\cos\dfrac{\theta}{2}$

$BO + OH = a\cos\dfrac{\theta}{2}$

$BO = a\cos\dfrac{\theta}{2} - r$

En el ◺BTO: $\text{sen}\dfrac{\theta}{2} = \dfrac{OT}{BO} = \dfrac{r}{a\cos\dfrac{\theta}{2} - r}$

$a\text{sen}\dfrac{\theta}{2}\cos\dfrac{\theta}{2} - r\text{sen}\dfrac{\theta}{2} = r$

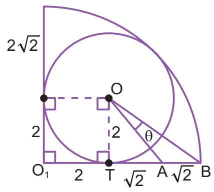
$a\text{sen}\dfrac{\theta}{2}\cos\dfrac{\theta}{2} = r\left(1 + \text{sen}\dfrac{\theta}{2}\right)$

$\Rightarrow r = \dfrac{a\text{sen}\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}{1 + \text{sen}\dfrac{\theta}{2}} = \dfrac{a\cos\dfrac{\theta}{2}}{\dfrac{1}{\text{sen}\dfrac{\theta}{2}} + \dfrac{\text{sen}\dfrac{\theta}{2}}{\text{sen}\dfrac{\theta}{2}}}$

$\therefore r = \dfrac{a\cos\dfrac{\theta}{2}}{1 + \csc\dfrac{\theta}{2}}$

Clave A

10.



Por el teorema de Pitágoras:
$OA = \sqrt{6} \quad \wedge \quad OB = 2\sqrt{3}$

En el $\triangle$OAB, igualando áreas:

$$\frac{(\text{base})(\text{altura})}{2} = \frac{(OA)(OB)}{2}\text{sen}\theta$$

$$\frac{(\sqrt{2})(2)}{2} = \frac{(\sqrt{6})(2\sqrt{3})}{2}.\text{sen}\theta$$

$$\Rightarrow \text{sen}\theta = \frac{2\sqrt{2}}{6\sqrt{2}} = \frac{1}{3}$$

Como: $\text{sen}\theta \,.\, \csc\theta = 1$

$$\left(\frac{1}{3}\right).\csc\theta = 1$$

$$\therefore \csc\theta = 3$$

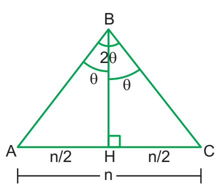
Clave B

11. • ⊿ AEB: $AE = AB\text{sen}\theta$
$AE = m\text{sen}\theta$
• ⊿ CFD: $FC = CD\text{sen}\theta$
$FC = m\text{sen}\theta$
• ⊿ ADC: $m\angle CAD = \theta$
$AC = CD\csc\theta$
$AC = m\csc\theta$

Luego: $AC = AE + EF + FC$
$m\csc\theta = m\text{sen}\theta + EF + m\text{sen}\theta$
$\therefore EF = m\csc\theta - 2m\text{sen}\theta$

Clave C

12. Del dato el $\triangle$ABC es isósceles:



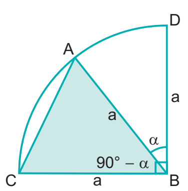
Por dato: AC = n
Entonces, la bisectriz trazada desde el ángulo B, también es altura y mediana. Luego:

$BC = \frac{n}{2}\csc\theta = AB$

Piden el perímetro del $\triangle$ABC:
$2p = AB + BC + AC$
$2p = \frac{n}{2}\csc\theta + \frac{n}{2}\csc\theta + n$
$\therefore 2p = n\csc\theta + n = n(\csc\theta + 1)$

Clave A

13.



$$\Rightarrow \text{Área} = \frac{a\,.\,a\,.\,\text{sen}(90° - \alpha)}{2}$$

$$\therefore \text{Área} = 0{,}5a^2\cos\alpha$$

Clave E

14. $A\square\,ABNM = A\triangle ABC - A\triangle MNC$

$$A\square\,ABNM = \frac{3a\,.\,2b}{2}\text{sen}\alpha - \frac{a\,.\,b}{2}\text{sen}\alpha$$

$$A\square\,ABNM = 3ab\text{sen}\alpha - \frac{ab}{2}\text{sen}\alpha$$

$$A\square\,ABNM = \frac{5}{2}ab\text{sen}\alpha$$

$$\therefore A\square\,ABNM = 2{,}5ab\text{sen}\alpha$$

Clave D

PRACTIQUEMOS

Nivel 1 (página 37) Unidad 2

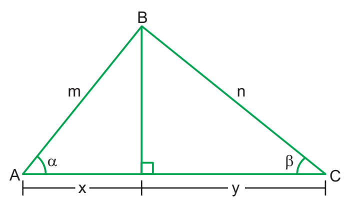
Comunicación matemática

1.

2.

Razonamiento y demostración

3.



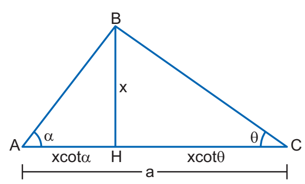
$\frac{x}{m} = \cos\alpha \Rightarrow x = m\cos\alpha$

$\frac{y}{n} = \cos\beta \Rightarrow y = n\cos\beta$

$\therefore AC = x + y = m\cos\alpha + n\cos\beta$

Clave B

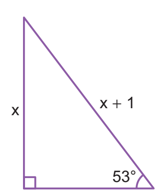
4.



$AC = AH + HC$
$a = x\cot\alpha + x\cot\theta$
$\therefore x(\cot\alpha + \cot\theta) = a$

Clave A

5.



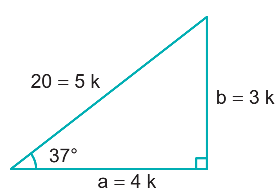
$\text{sen}53° = \frac{x}{x+1} = \frac{4}{5}$

$5x = 4x + 4$

$\therefore x = 4$

Clave D

6.

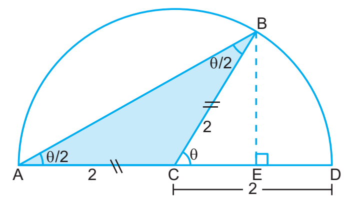


$5k = 20$
$\Rightarrow k = 4$

$\Rightarrow a + b = 4k + 3k$
$a + b = 7k = 7(4)$
$\therefore a + b = 28$

Clave B

7.



En el $\triangle$BEC:

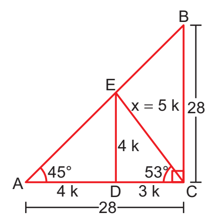
$\frac{BE}{2} = \text{sen}\theta \Rightarrow BE = 2\text{sen}\theta$

$A_{\triangle ABC} = \frac{AC\,.\,BE}{2} = \frac{2\,.\,2\text{sen}\theta}{2}$

$\therefore A_{\triangle ABC} = 2\text{sen}\theta$

Clave A

8.

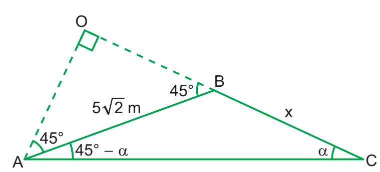


$4k + 3k = 28$
$7k = 28$
$\Rightarrow k = 4$
$\therefore x = 5k = 5(4) = 20$

Clave B

Resolución de problemas

9.



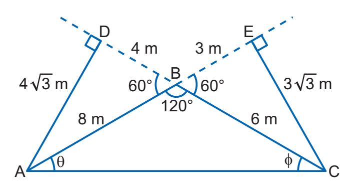
• $AO = 5\sqrt{2}\,\text{sen}45°$
$AO = 5\text{ m} \Rightarrow OB = 5\text{ m}$

• $\tan\alpha = \frac{AO}{CO} = \frac{5}{x+5} \Rightarrow \frac{5}{12} = \frac{5}{x+5}$

$\therefore x = 7\text{ m}$

Clave D

10.



• En ⊿ ADC:

$\cot\phi = \frac{10}{4\sqrt{3}} = \frac{5\sqrt{3}}{6}$

• En ⊿ AEC:

$\tan\theta = \frac{3\sqrt{3}}{11}$

$\therefore \cot\phi\,.\,\tan\theta = \frac{15}{22}$

Clave C

Nivel 2 (página 38) Unidad 2

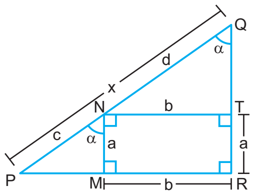
Comunicación matemática

11.

12.

Razonamiento y demostración

13.



En el ⊿PMN:

$\frac{c}{a} = \sec\alpha \Rightarrow c = a\sec\alpha$

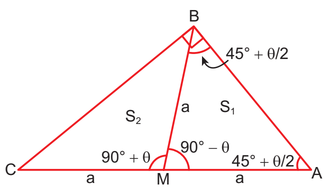
En el ⊿NTQ:

$\frac{d}{b} = \csc\alpha \Rightarrow d = b\csc\alpha$

$\therefore x = c + d = a\sec\alpha + b\csc\alpha$

Clave D

14.



$$\left.\begin{array}{l} m\angle A - m\angle C = \theta \\ m\angle A + m\angle C = 90° \end{array}\right] (+)$$

$m\angle A = 45° + \frac{\theta}{2}$

$S_1 = \frac{a\,.\,a}{2}.\text{sen}(90° - \theta) = a^2.\frac{\cos\theta}{2}$

$S_2 = \frac{a\,.\,a}{2}.\text{sen}(90° + \theta) = a^2.\frac{\cos\theta}{2}$

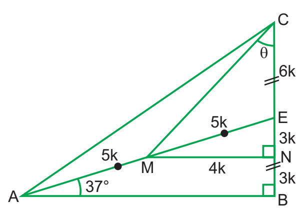
$S_{TOTAL} = S_1 + S_2 = \frac{a^2.\cos\theta}{2} + \frac{a^2.\cos\theta}{2}$

$\therefore\ S_{TOTAL} = a^2\cos\theta$

Clave B

15. Hacemos: CE = 6k

Además: MN es base media del △AEB.

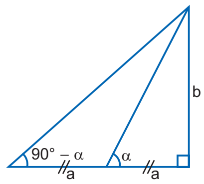


$\Rightarrow \tan\theta = \frac{4k}{9k}$

$\therefore \tan\theta = \frac{4}{9}$

Clave C

16.



$\tan\alpha = \frac{b}{a} = \frac{2a}{b}$ ...(1)

Luego: $b^2 = 2a^2$

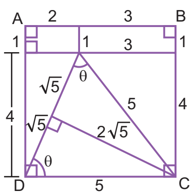
$\Rightarrow b = \sqrt{2}\,a$

Reemplazando el valor de b en (1):

$\therefore \tan\alpha = \frac{\sqrt{2}\,a}{a} = \sqrt{2}$

Clave A

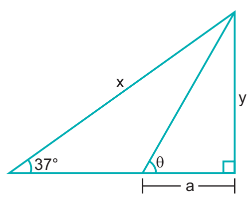
17.



$\therefore \text{sen}\theta = \frac{2\sqrt{5}}{5}$

Clave C

18.



$\frac{y}{a} = \tan\theta$

$y = a\tan\theta$

$\frac{y}{x} = \text{sen}37° = \frac{3}{5}$

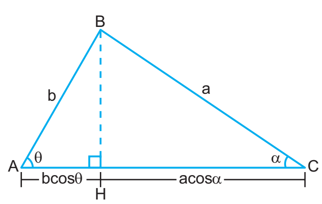
$\Rightarrow x = \frac{5}{3}y$

$\therefore x = \frac{5}{3}a\tan\theta$

Clave B

Resolución de problemas

19.



$\Rightarrow\ b\cos\theta + a\cos\alpha = AC$

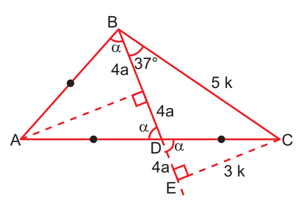
Dato:

$b\cos\theta + a\cos\alpha = 4$

$\therefore\ AC = 4$

Clave A

20.



En el ⊿CEB:

$12a = 4k \Rightarrow\ k = 3a$

$\therefore\ \tan\alpha = \frac{3k}{4a} = \frac{3(3a)}{4a} = \frac{9}{4}$

Clave E

Nivel 3 (página 39) Unidad 2

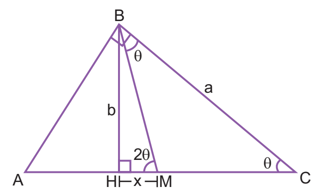
Comunicación matemática

21.

22.

Razonamiento y demostración

23.



Propiedad:

$\overline{BM}$: mediana

$\frac{b}{a} = \text{sen}\theta$

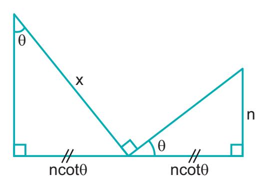
$\Rightarrow b = a\,\text{sen}\theta$

$\frac{x}{b} = \cot 2\theta$

$x = b\cot 2\theta = a\,\text{sen}\theta\cot 2\theta$

Clave E

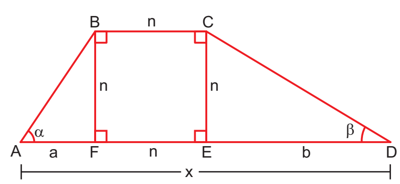
24.



$\frac{x}{n\cot\theta} = \csc\theta \quad \therefore x = n\cot\theta\csc\theta$

Clave B

25.



$\tan\alpha = \frac{n}{a} \ \wedge\ \tan\beta = \frac{n}{b}$

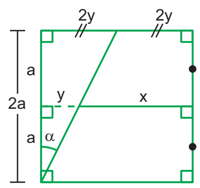
$\Rightarrow a = n\cot\alpha \ \wedge\ b = n\cot\beta$

$\Rightarrow x = a + b + n = n\cot\alpha + n\cot\beta + n$

$\therefore x = n(\cot\alpha + \cot\beta + 1)$

Clave C

26.

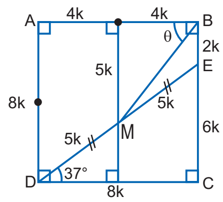


En el gráfico:
$y = a\tan\alpha$

Además:
$y + x = 4y$
$\therefore x = 3y = 3a\tan\alpha$

Clave D

27.



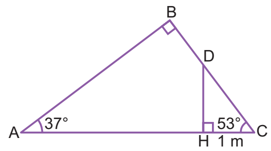
$\Rightarrow \tan\theta = \dfrac{5k}{4k}$

$\therefore \tan\theta = \dfrac{5}{4}$

Clave E

Resolución de problemas

28.

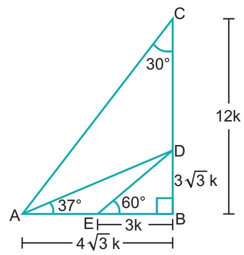


$DC = 1\sec 53°$

$\therefore DC = \dfrac{5}{3}$ m

Clave D

29.



Del gráfico:

$CD = 12k - 3\sqrt{3}\ k$

$CD = k(12 - 3\sqrt{3})$

$AE = AB - EB$

$2 - \dfrac{\sqrt{3}}{2} = 4\sqrt{3}k - 3k$

$\dfrac{4-\sqrt{3}}{2} = k\sqrt{3}\left(4-\sqrt{3}\right) \Rightarrow k = \dfrac{1}{2\sqrt{3}}$

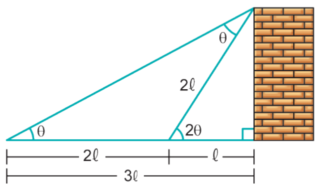
$\therefore CD = \dfrac{1}{2\sqrt{3}}\left(12 - 3\sqrt{3}\right)$

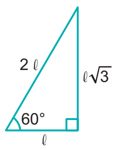
$CD = 2\sqrt{3} - \dfrac{3}{2}$

Clave E

APLICAMOS LO APRENDIDO
(página 40) Unidad 2

1.





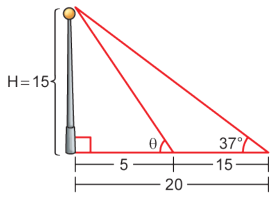
Triángulo notable de 30° y 60°:

$\Rightarrow 2\theta = 60°$

$\therefore \theta = 30°$

Clave B

2.

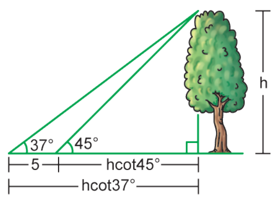


$\Rightarrow \tan\theta = \dfrac{H}{5} = \dfrac{15}{5}$

$\therefore \tan\theta = 3$

Clave C

3.



Del gráfico:

$h\cot 37° = 5 + h\cot 45°$

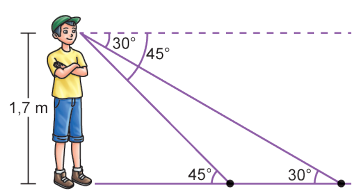
$\Rightarrow h\left(\dfrac{4}{3}\right) = 5 + h(1)$

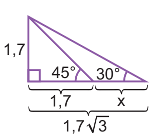
$\dfrac{4h}{3} = 5 + h \Rightarrow \dfrac{h}{3} = 5$

$\therefore\ h = 15$ m

Clave D

4.





Del gráfico:

$1{,}7 + x = 1{,}7\sqrt{3} = 2{,}94$

$x = 2{,}94 - 1{,}7$

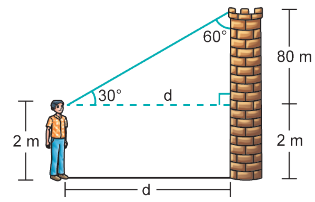
$x = 1{,}24$

La distancia al grano más lejano $= 1{,}7\sqrt{3}$ m

La distancia entre granos $= 1{,}24$ m

Clave E

5.



Piden: la distancia (d) de la base de la torre hacia la persona.

Del gráfico:

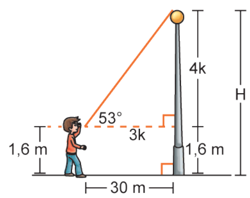
$\cot 30° = \dfrac{d}{80}$

$\Rightarrow d = 80\cot 30° = 80 \cdot \left(\dfrac{\sqrt{3}}{1}\right)$

$\therefore d = 80\sqrt{3}$ m

Clave E

6.



Del gráfico:

$3k = 30$

$k = 10$

Luego:

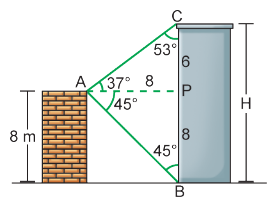
La altura del poste: H

$H = 4k + 1{,}6 = 4(10) + 1{,}6 = 41{,}6$

$\therefore H = 41{,}6$ m

Clave C

7.



Del ◺APB, notable de 45°:

PB = AP = 8

Del ◺APC, notable de 37° y 53°:

AP = 8 ∧ CP = 6

Sea H: la altura del edificio.

Del gráfico:

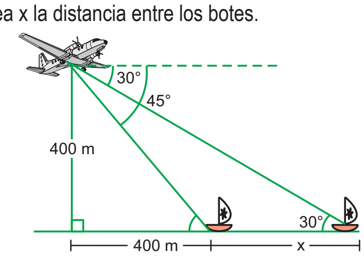
$H = CP + PB$

$H = 6 + 8 = 14$

$\therefore H = 14$ m

Clave A

8. Sea x la distancia entre los botes.



Del gráfico:

$\dfrac{x + 400}{400} = \cot 30°$

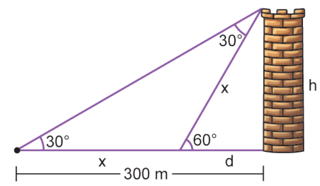
$x + 400 = \left(\sqrt{3}\right) . 400$

$x = 400\sqrt{3} - 400$

$\therefore x = 400(\sqrt{3} - 1)$ m

Clave A

9.



Del gráfico: $h = 300\tan 30°$

$h = 300\dfrac{\sqrt{3}}{3} = 100\sqrt{3}$ m

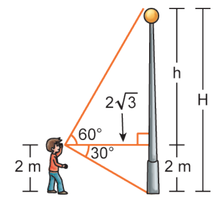
Además: $d = 100$ m

Luego: $x = 2d$

$\therefore\ x = 2(100) = 200$ m

Clave E

10.



Del gráfico:

$h = 2\sqrt{3}\tan 60°$

$h = 2\sqrt{3}\sqrt{3}$

$h = 2\sqrt{9}$

$h = 6$

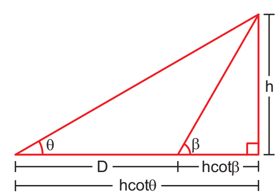
Luego: $H = h + 2$

$H = 6 + 2$

$\therefore\ H = 8$ m

Clave A

11.



Del gráfico:

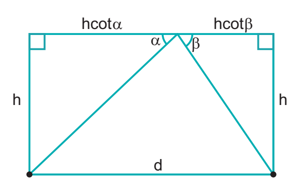
$h\cot\theta = D + h\cot\beta$

$h(\cot\theta - \cot\beta) = D$

$h = \dfrac{D}{\cot\theta - \cot\beta}$

Clave D

12.



Del gráfico

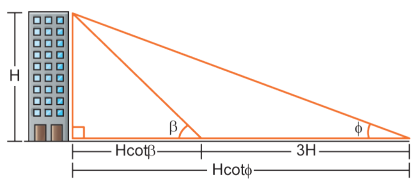
$d = h\cot\alpha + h\cot\beta$

$d = h(\cot\alpha + \cot\beta)$

$h = \dfrac{d}{\cot\alpha + \cot\beta}$

Clave C

13.



Piden: $E = \cot\phi - \cot\beta$

Del gráfico: $H\cot\phi = H\cot\beta + 3H$

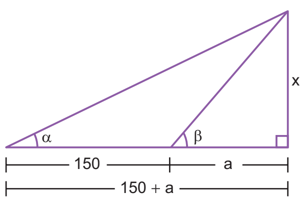
$H(\cot\phi - \cot\beta) = 3H$

$$\underbrace{\cot\phi - \cot\beta}_{} = \frac{3H}{H}$$

$\therefore\ E = 3$

Clave C

14.



$$\cot\beta = \frac{a}{x} \quad \wedge \quad \cot\alpha = \frac{a+150}{x}$$

$$\cot\alpha - \cot\beta = \frac{1}{3}$$

$$\frac{a+150}{x} - \frac{a}{x} = \frac{1}{3}$$

$$\frac{150}{x} = \frac{1}{3} \Rightarrow x = 450 \text{ m}$$

Clave C

PRACTIQUEMOS

Nivel 1 (página 42) Unidad 2

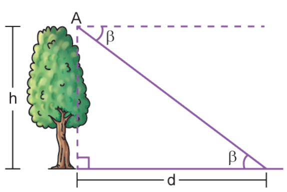
Comunicación matemática

1.

2.

Razonamiento y demostración

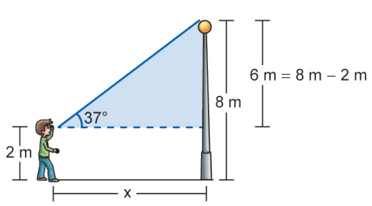
3.

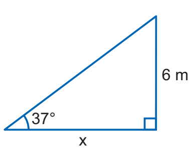


$\therefore\ h = d\tan\beta$

Clave A

4.





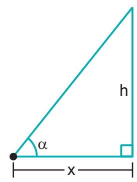
$x = 6\cot 37°$

$\therefore\ x = 8$ m

Clave E

Resolución de problemas

5. Sea h la altura del poste.

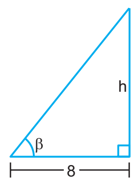


$$\frac{x}{h} = \cot\alpha$$

$\therefore\ x = h\cot\alpha$

Clave D

6. Sea h la altura del poste.

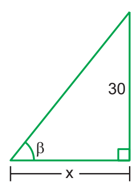


$$\frac{h}{8} = \tan\beta$$

$\Rightarrow h = 8\tan\beta$

Clave A

7.



$$\frac{x}{30} = \cot\beta$$

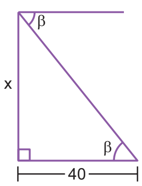
Por dato:

$$\cot\beta = \frac{5}{2}$$

$$\therefore x = 30 \cdot \frac{5}{2} = 75 \text{ m}$$

Clave C

8. Sea x la altura del edificio.



Por dato:

$$\tan\beta = \frac{5}{2}$$

$$\tan\beta = \frac{x}{40}$$

$$\Rightarrow \frac{x}{40} = \frac{5}{2} \quad \therefore x = 100 \text{ m}$$

Clave A

Nivel 2 (página 42) Unidad 2

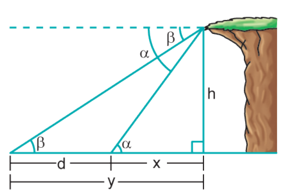
Comunicación matemática

9.

10.

Razonamiento y demostración

11.



$$\frac{h}{y} = \tan\beta \ \wedge \ \frac{h}{x} = \tan\alpha$$

$\Rightarrow y = h\cot\beta \ \wedge \ x = h\cot\alpha$

Como:

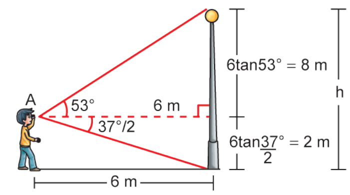
$d + x = y \Rightarrow d + h\cot\alpha = h\cot\beta$

Despejando h, tenemos:

$$\therefore\ h = \frac{d}{\cot\beta - \cot\alpha}$$

Clave A

12.

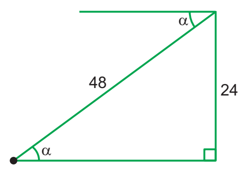


$\therefore\ h = 8 \text{ m} + 2 \text{ m} = 10$ m

Clave E

Resolución de problemas

13.



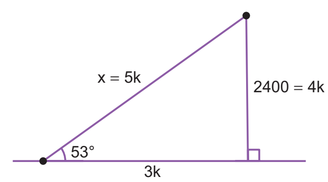
$$\text{sen}\alpha = \frac{24}{48}$$

$$\text{sen}\alpha = \frac{1}{2} \Rightarrow \alpha = \text{arcsen}\left(\frac{1}{2}\right)$$

$\therefore \alpha = 30°$

Clave C

14.

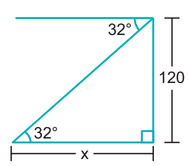


$4k = 2400 \Rightarrow k = 600$

$\therefore x = 5k = 5(600) = 3000$ m

Clave B

15. Sea x la distancia de la base de la montaña al objeto.



$$\frac{x}{120} = \cot 32°$$

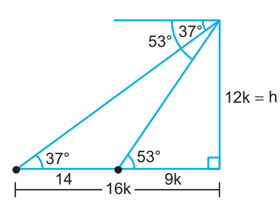
$\Rightarrow x = 120(\cot 32°)$

$x = 120(1{,}60033)$

$\therefore x = 192{,}04$ m

Clave B

16. Sea h la altura del faro.



$14 + 9k = 16k$

$14 = 7k$

$\Rightarrow k = 2$

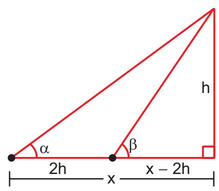
$h = 12k$

$h = 12(2)$

$\therefore h = 24$ m

Clave C

17.



$\cot\alpha = \frac{x}{h} \quad \wedge \quad \cot\beta = \frac{x-2h}{h}$

$\cot\alpha - \cot\beta = \frac{x}{h} - \frac{(x-2h)}{h} = \frac{2h}{h} = 2$

$\therefore P = 2$

Clave B

Nivel 3 (página 43) Unidad 2

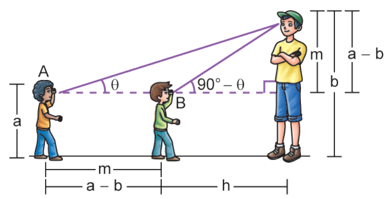
Comunicación matemática

18.

19.

Razonamiento y demostración

20.



$\tan\theta = \frac{h}{a-b} \wedge \cot\theta = \frac{h+a-b}{a-b}$

$\cot\theta - \tan\theta = \frac{h+a-b-h}{a-b} = 1$

$\Rightarrow [\cot\theta - \tan\theta]^2 = (1)^2$

$\Rightarrow \cot^2\theta + \tan^2\theta = 3$

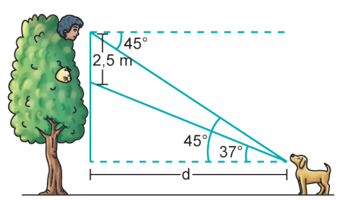
$H = \cot\theta + \tan\theta$; $H^2 = \cot^2\theta + \tan^2\theta + 2$

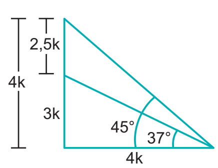
$H^2 = 3 + 2$

$\therefore H = \sqrt{5}$

Clave A

21.





$\Rightarrow 3k + 2,5 = 4k$

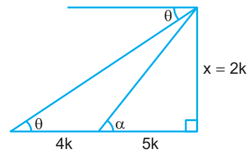
$k = 2,5$

$\therefore d = 4 . k = 10$ m

Clave A

Resolución de problemas

22. Sea x la altura de la colina.



$\tan\alpha = 0,4 = \frac{2}{5}$

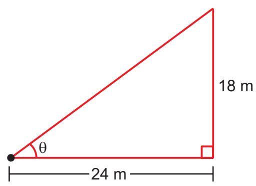
$\tan\theta = \frac{2}{9}$

$4k = 300 \Rightarrow k = 75$

Piden: $x = 2k = 2(75) = 150$ m

Clave C

23.



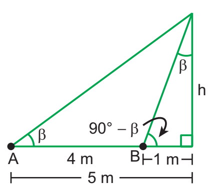
Nota:
Notación de la función trigonométrica inversa:
$FT(\alpha) = N \Rightarrow \alpha = \text{arc } FT(N)$
Se lee: α es un arco cuya función trigonométrica es N.
En el problema:

$\tan\theta = \frac{18}{24} = \frac{3}{4}$

$\therefore \theta = \arctan\left(\frac{3}{4}\right) = 37°$

Clave A

24.

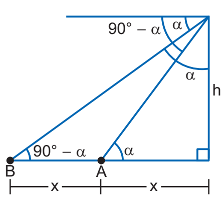


$\tan\beta = \frac{h}{5} = \frac{1}{h} \Rightarrow h^2 = 5 \Rightarrow h = \sqrt{5}$

$\therefore \cot\beta = \frac{5}{h} = \frac{5}{\sqrt{5}} = \frac{5}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \sqrt{5}$

Clave D

25.



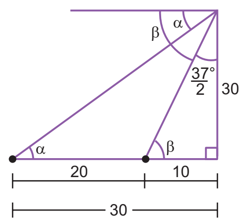
$\tan\alpha = \frac{h}{x} = \frac{2x}{h}$

$h^2 = 2x^2 \Rightarrow h = \sqrt{2}\,x$

$\therefore \cot\alpha = \frac{x}{h} = \frac{x}{\sqrt{2}\,x} = \frac{\sqrt{2}}{2}$

Clave E

26.



Piden: $\beta - \alpha$

$\tan\beta = \frac{30}{10} = 3 \quad \wedge \quad \tan\alpha = \frac{30}{30} = 1$

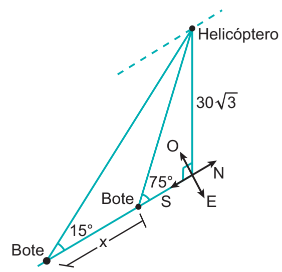
$\beta = \arctan(3) \qquad \alpha = \arctan 1$

$\Rightarrow \beta = \frac{143°}{2} \qquad \Rightarrow \alpha = 45°$

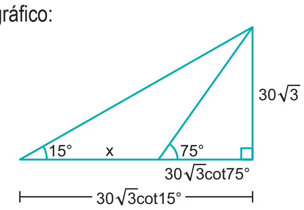
$\beta - \alpha = \frac{143°}{2} - 45° = \frac{53°}{2} = 26,5°$

Clave A

27.



Del gráfico:



$x + 30\sqrt{3}\cot 75° = 30\sqrt{3}\cot 15°$

$\Rightarrow x = 30\sqrt{3}(\cot 15° - \cot 75°)$

$x = 30\sqrt{3}(2+\sqrt{3} - (2-\sqrt{3}))$

$x = 30\sqrt{3}(2\sqrt{3})$

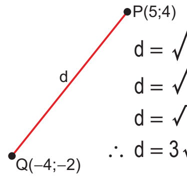
$\therefore x = 180$ m

Clave C

SISTEMA CARTESIANO

APLICAMOS LO APRENDIDO
(página 45) Unidad 2

1.



$d = \sqrt{(5-(-4))^2 + (4-(-2))^2}$

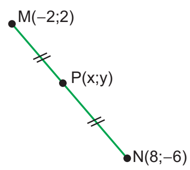
$d = \sqrt{9^2 + 6^2}$

$d = \sqrt{117}$

$\therefore\ d = 3\sqrt{13}$

Clave E

2.



- $x = \dfrac{-2+8}{2}$

$\Rightarrow\ x = 3$

- $y = \dfrac{2-6}{2}$

$\Rightarrow\ y = -2$

$\therefore\ P(x;\ y) = P(3;\ -2)$

Clave B

3. Si G(x; y) es baricentro, entonces:

$$x = \frac{x_1 + x_2 + x_3}{3} \ \wedge\ y = \frac{y_1 + y_2 + y_3}{3}$$

Reemplazamos:

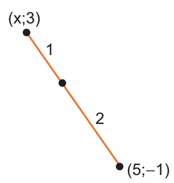
$$x = \frac{5 + 1 + (-3)}{3} \Rightarrow x = 1$$

$$y = \frac{6 + (-4) + 7}{3} \Rightarrow y = 3$$

$\therefore\ G(x;\ y) = G(1;\ 3)$

Clave A

4. Se debe considerar el siguiente gráfico:



Entonces:

$$\frac{2(x) + 1(5)}{2+1} = \frac{11}{3}$$

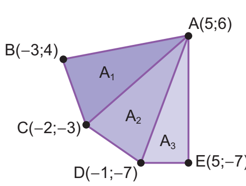
$$\frac{2x+5}{3} = \frac{11}{3}$$

$$2x = 6$$

$$\therefore x = 3$$

Clave B

5.



$$\begin{array}{r|rr|r} & 5 & 6 & \\ -18 & -3 & 4 & 20 \\ -8 & -2 & -3 & 9 \\ -15 & 5 & 6 & -12 \\ \hline -41 & & & 17 \end{array}$$

$$\Rightarrow A_1 = \frac{|17-(-41)|}{2}$$

$$A_1 = \frac{|58|}{2} = \frac{58}{2} \Rightarrow A_1 = 29$$

-

$$\begin{array}{r|rr|r} & 5 & 6 & \\ -12 & -2 & -3 & -15 \\ 3 & -1 & -7 & 14 \\ -35 & 5 & 6 & -6 \\ \hline -44 & & & -7 \end{array}$$

$$\Rightarrow A_2 = \frac{|-7-(-44)|}{2}$$

$$A_2 = \frac{|37|}{2} = \frac{37}{2}$$

$A_2 = 18{,}5$

-

$$\begin{array}{r|rr|r} & 5 & 6 & \\ -6 & -1 & -7 & -35 \\ -35 & 5 & -7 & 7 \\ -35 & 5 & 6 & 30 \\ \hline -76 & & & 2 \end{array}$$

$$\Rightarrow A_3 = \frac{|2-(-76)|}{2}$$

$$A_3 = \frac{|78|}{2} = \frac{78}{2}$$

$A_3 = 39$

Piden el área total: A_T

$A_T = A_1 + A_2 + A_3$

$A_T = (29) + (18{,}5) + (39)$

$\therefore A_T = 86{,}5$

Clave A

6.

-

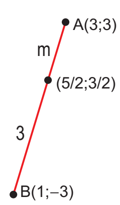
$$\begin{array}{r|rr|r} & -1 & 2 & \\ 6 & 3 & -4 & 4 \\ -16 & 4 & 1 & 3 \\ -1 & -1 & 2 & 8 \\ \hline -11 & & & 15 \end{array}$$

$$S = \frac{|15-(-11)|}{2} = \frac{26}{2}$$

$\therefore\ S = 13$

Clave A

7.



Entonces:

$$\frac{3(3) + m(1)}{3+m} = \frac{5}{2}$$

$$\frac{9+m}{3+m} = \frac{5}{2}$$

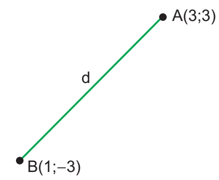
$18 + 2m = 15 + 5m$

$3 = 3m$

$\Rightarrow m = 1$

Clave C

8.



$d = \sqrt{(3-1)^2 + [3-(-3)]^2}$

$d = \sqrt{2^2 + 6^2} = \sqrt{40}$

$\therefore d = 2\sqrt{10}$

Clave D

9. Sea G(x; y) baricentro del triángulo PQR.

$$G(x;\ y) = \frac{P+Q+R}{3}$$

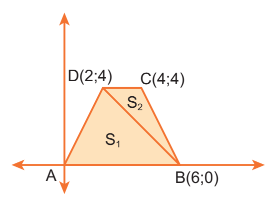
$$G(x;\ y) = \frac{(1;1) + (-4;6) + (0;5)}{3}$$

$$G(x;\ y) = \frac{(1-4+0;1+6+5)}{3} = \frac{(-3;12)}{3}$$

$\therefore\ G(x;\ y) = (-1;\ 4)$

Clave C

10.
- Dividimos el trapecio en dos triángulos: ADB y DCB



$\Rightarrow S = S_1 + S_2$

- S_1:

$$\begin{array}{r|rr|r} & 0 & 0 & \\ 0 & 6 & 0 & 0 \\ 0 & 2 & 4 & 24 \\ 0 & 0 & 0 & 0 \\ \hline 0 & & & 24 \end{array}$$

$$\Rightarrow S_1 = \frac{|24-0|}{2} = \frac{24}{2}$$

$S_1 = 12$

- S_2:

$$\begin{array}{r|rr|r} & 2 & 4 & \\ 24 & 6 & 0 & 0 \\ 0 & 4 & 4 & 24 \\ 8 & 2 & 4 & 16 \\ \hline 32 & & & 40 \end{array}$$

$$\Rightarrow\ S_2 = \frac{|40-32|}{2} = \frac{8}{2}$$

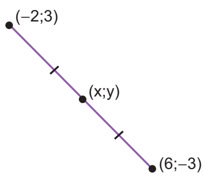
$S_2 = 4$

$\Rightarrow\ S = S_1 + S_2 = 12 + 4$

$\therefore\ S = 16$

Clave D

11.



Entonces:

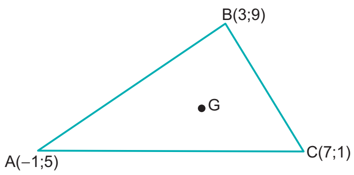
$$(x; y) = \frac{(-2; 3) + (6; -3)}{2}$$

$$(x; y) = \left(\frac{-2+6}{2}; \frac{3-3}{2}\right)$$

$\therefore (x; y) = (2; 0)$

Clave C

12.



$$G = \frac{A+B+C}{3}$$

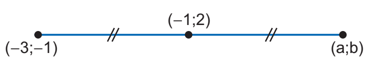
$$G = \frac{(-1; 5) + (7; 1) + (3; 9)}{3}$$

$$G = \left(\frac{7+3-1}{3}; \frac{5+9+1}{3}\right)$$

$G = (3; 5)$

Clave C

13.



$$\frac{-3+a}{2} = -1 \quad \wedge \quad \frac{b-1}{2} = 2$$

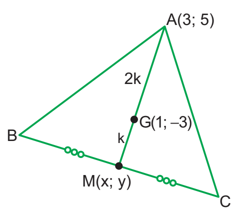
$$-3 + a = -2 \qquad b - 1 = 4$$

$$\Rightarrow a = 1 \qquad \Rightarrow b = 5$$

$\therefore a + b = 1 + 5 = 6$

Clave D

14.



Por dato: G es baricentro del $\triangle ABC$.

Entonces: M(x; y) es punto medio de $\overline{BC}$.

Del gráfico: G(1; – 3) divide al segmento AM en la razón $r = \frac{MG}{GA} = \frac{k}{2k} \Rightarrow r = \frac{1}{2}$

$$\Rightarrow 1 = \frac{x + \frac{1}{2}(3)}{1 + \frac{1}{2}} \Rightarrow x = 0$$

$$\Rightarrow -3 = \frac{y + \frac{1}{2}(5)}{1 + \frac{1}{2}} \Rightarrow y = -7$$

$\Rightarrow M(x; y) = M(0; -7)$

Piden: la suma de coordenadas de M.

$x + y = 0 + (-7) = -7$

$\therefore x + y = -7$

Clave C

PRACTIQUEMOS

Nivel 1 (página 47) Unidad 2

Comunicación matemática

1. Por definición:

Ia

IIc

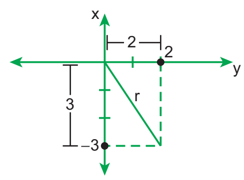
IIIb

Clave E

2.

Razonamiento y demostración

3.



$x = 2 \quad \wedge \quad y = -3$

$r^2 = x^2 + y^2$

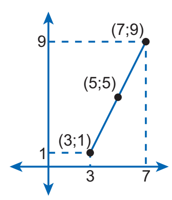
$r^2 = 2^2 + (-3)^2$

$r^2 = 4 + 9 = 13$

$r = \sqrt{13}$

Clave C

4.



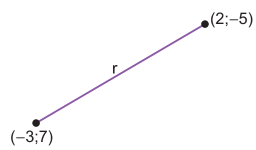
$x = 5 \;\wedge\; y = 5$

$\Rightarrow r^2 = 5^2 + 5^2 = 50$

$\Rightarrow r = 5\sqrt{2}$

Clave C

5.



$r^2 = (2 - (-3))^2 + (-5 - 7)^2$

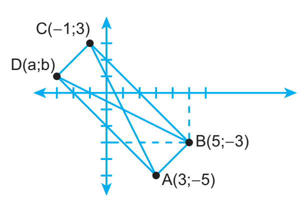
$r^2 = 5^2 + (-12)^2 = 169$

$\Rightarrow r = 13$

$\therefore$ Diámetro $= 2r = 2(13) = 26$

Clave C

6.



$$\frac{A+C}{2} = \frac{B+D}{2}$$

$(3; -5) + (-1; 3) = (5; -3) + (a; b)$

$(2; -2) = (5 + a; b - 3)$

$\Rightarrow 5 + a = 2 \quad \wedge \quad b - 3 = -2$

$a = -3 \quad \wedge \quad b = 1$

$\therefore D = (-3; 1)$

Clave C

7.

$$x = \sqrt{(1-(-2))^2 + (3-(-4))^2}$$

$$x = \sqrt{3^2 + 7^2}$$

$$x = \sqrt{58}$$

Clave B

8. Por dato, los vértices del cuadrilátero ABCD son: A(–5; 6), B(–2; 7), C(0; 1) y D(–3; 0)

Luego:

- $AB = d_{AB}$

$$d_{AB} = \sqrt{(-5-(-2))^2 + (6-7)^2}$$

$$\Rightarrow d_{AB} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{10}$$

$$\Rightarrow AB = \sqrt{10}$$

- $BC = d_{BC}$

$$d_{BC} = \sqrt{(-2-0)^2 + (7-1)^2}$$

$$\Rightarrow d_{BC} = \sqrt{(-2)^2 + (6)^2} = \sqrt{40}$$

$$\Rightarrow BC = 2\sqrt{10}$$

- $CD = d_{CD}$

$$d_{CD} = \sqrt{(0-(-3))^2 + (1-0)^2}$$

$$\Rightarrow d_{CD} = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$$

$$\Rightarrow CD = \sqrt{10}$$

- $DA = d_{DA}$

$$d_{DA} = \sqrt{(-3-(-5))^2 + (0-6)^2}$$

$$\Rightarrow d_{DA} = \sqrt{(2)^2 + (-6)^2} = \sqrt{40}$$

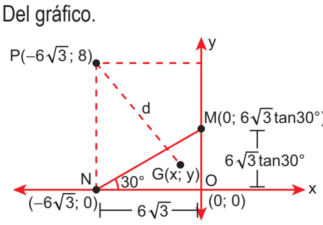
$$\Rightarrow DA = 2\sqrt{10}$$

Por lo tanto, el mayor lado mide $2\sqrt{10}$.

Clave B

Resolución de problemas

9. - Del gráfico.



$OM = 6\sqrt{3}\ \tan 30°$

$OM = 6\sqrt{3}\left(\frac{\sqrt{3}}{3}\right) = 6$

$\Rightarrow M(x; y) = (0, 6)$

$N(x; y) = (-6\sqrt{3}; 0)$
$0(x; y) = (0; 0)$

- Hallamos el punto G(x; y)

$G(x; y) = \dfrac{M + N + O}{3}$

$G(x; y) = [(0; 6) + (-6\sqrt{3}; 0) + (0; 0)]/3$

$G(x; y) = \dfrac{(-6\sqrt{3}; 6)}{3} = (-2\sqrt{3}; 2)$

- La distancia PG:

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$d = \sqrt{(-6\sqrt{3} - (-2\sqrt{3}))^2 + (8 - 2)^2}$

$d = \sqrt{(-4\sqrt{3})^2 + 6^2} = \sqrt{48 + 36}$

$d = \sqrt{84} \Rightarrow d = 2\sqrt{21}$

Clave B

10. ▪ $\overline{AC}$ es diagonal:

$d = \sqrt{(0 - 3)^2 + (0 - 4)^2}$

$d = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

▪ Hallamos AB y AD:

$AB = \sqrt{(0 - b_1)^2 + (0 - b_2)^2}$

$d\dfrac{\sqrt{2}}{2} = \sqrt{b_1^2 + b_2^2}$

$\therefore \dfrac{25}{2} = b_1^2 + b_2^2$

$AD = \sqrt{(0 - d_1)^2 + (0 - d_2)^2}$

$d\dfrac{\sqrt{2}}{2} = \sqrt{d_1^2 + d_2^2}$

$\therefore \dfrac{25}{2} = d_1^2 + d_2^2$

▪ Hallamos la diagonal $\overline{BD}$:

$BD = \sqrt{(b_1 - d_1)^2 + (b_2 - d_2)^2}$

$5^2 = b_1^2 + d_1^2 - 2b_1d_1 + b_2^2 + d_2^2 - 2b_2d_2$

$25 = \dfrac{25}{2} + \dfrac{25}{2} - 2(b_1d_1 + b_2d_2)$

$25 = 25 - 2(b_1d_1 + b_2d_2)$

$\therefore\ b_1d_1 + b_2d_2 = k = 0$

Clave B

Nivel 2 (página 47) Unidad 2

Comunicación matemática

11. I. En la fórmula del baricentro es necesario tener los 3 vértices.

$\therefore$ I (F)

II. El radio vector es la distancia de un punto al origen del sistema.

$\therefore$ II (F)

III. En el IC el punto P es de la forma.
P(+; +)

El producto es: (+) (+) = (+)

$\therefore$ III (V)

Clave C

12. M: Tenemos el punto medio:

$P(x; y) = \dfrac{A + B}{2}$

$P(x; y) = \dfrac{(3; -2) + (1; 4)}{2}$

$P(x; y) = (2; 1) \Rightarrow x = 2$
$y = 1$

$\therefore M = xy = 2$

N: Hallamos el lado del triángulo equilátero.

$l = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$l = \sqrt{(3 - (-1))^2 + (1 - 4)^2}$

$l = \sqrt{4^2 + (-3)^2} = \sqrt{25}$

$l = 5$

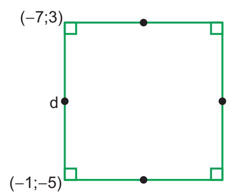
$\therefore N = 3l = 15$

$\Rightarrow 2N = 15M$

Clave A

Razonamiento y demostración

13.



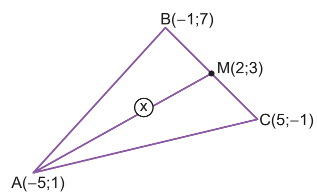
$d^2 = (-7 - (-1))^2 + (3 + 5)^2$

$d^2 = 36 + 64 \Rightarrow d = 10$

$\Rightarrow$ Perímetro = 4d = 4(10) = 40

Clave B

14.



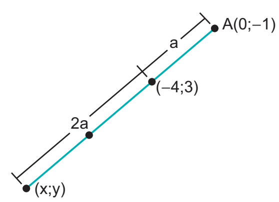
$x^2 = (-5 - 2)^2 + (1 - 3)^2$

$x^2 = (-7)^2 + (-2)^2 \Rightarrow x^2 = 53$

$\therefore x = \sqrt{53}$

Clave C

15.



$-4 = \dfrac{ax + 2a(0)}{3a} \Rightarrow x = -12$

$3 = \dfrac{ay + 2a(-1)}{3a} \Rightarrow 9 = y - 2$
$y = 11$

Por lo tanto, el punto es: (–12; 11)

Clave B

16. Sea M = (a; 0), tal que:

$\sqrt{(a - 2)^2 + (0 - (-3))^2} = 5$

$\Rightarrow (a - 2)^2 + 3^2 = 5^2$

$(a - 2)^2 + 9 = 25$

$(a - 2)^2 = 16$

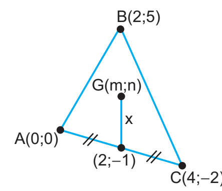
$\Rightarrow a - 2 = 4 \quad \vee \quad a - 2 = -4$

$a = 6 \quad \vee \quad a = -2$

$\Rightarrow M = (6; 0) \quad o \quad M = (-2; 0)$

Clave D

17.



$G = \dfrac{(0; 0) + (4; -2) + (2; 5)}{3}$

$G = (2; 1)$

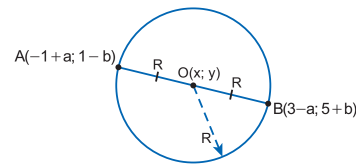
$x = \sqrt{(2 - 2)^2 + (1 - (-1))^2}$

$x = \sqrt{0^2 + 2^2}$

$\therefore x = 2$

Clave C

18.



Del gráfico: O es punto medio de $\overline{AB}$.

$\Rightarrow x = \dfrac{(-1 + a) + (3 - a)}{2} = \dfrac{2}{2} = 1$

$\Rightarrow x = 1$

$\Rightarrow y = \dfrac{(1 - b) + (5 + b)}{2} = \dfrac{6}{2} = 3$

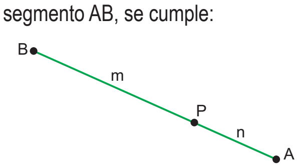
$\Rightarrow y = 3$

$\therefore O(x; y) = O(1; 3)$

Clave E

Resolución de problemas

19. En el segmento AB, se cumple:



$P = \dfrac{nB + mA}{n + m}$

Dato: $m = 2k \ \wedge\ n = k$

Reemplazamos:

$P(x; y) = \dfrac{k(-10; 6) + 2k(2; 3)}{k + 2k}$

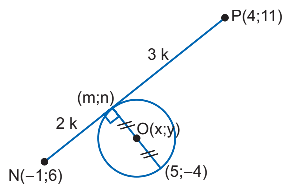
$$P(x;y)=\frac{k(-10;6)+k(4;6)}{3k}$$

$$P(x;y)=\frac{(-6;12)}{3}=(-2;4)$$

$\therefore\ x+y=2$

Clave C

20.



$$m=\frac{-1(3k)+4(2k)}{5k}=\frac{5k}{5k}\Rightarrow m=1$$

$$n=\frac{6(3k)+11(2k)}{5k}=\frac{40k}{5k}\Rightarrow n=8$$

$$\Rightarrow x=\frac{m+5}{2}=\frac{1+5}{2}\Rightarrow x=3$$

$$\Rightarrow y=\frac{n+-4}{2}=\frac{8-4}{2}\Rightarrow y=2$$

$\therefore\ O=(3;2)$

Clave D

Nivel 3 (página 48) Unidad 2

Comunicación matemática

21.

22.

a) De la ecuación; tenemos:

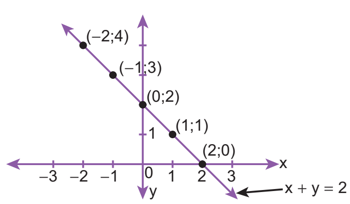
$x+y=2$

$y=2-x$

- Dando valores a x:

x	y
2	0
1	+1
0	+2
–1	+3
–2	+4

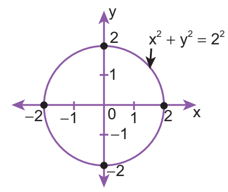
- Representemos los puntos en el plano y luego los unimos mediante una recta.



b) Extraemos la raíz de la ecuación.

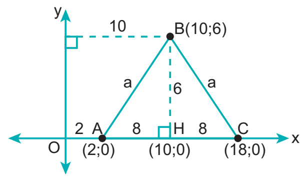
$$\sqrt{x^2+y^2}=\sqrt{4}$$

$$\sqrt{x^2+y^2}=2$$



Razonamiento y demostración

23.



Del gráfico: el $\triangle$ABC resulta isósceles.

En el $\triangle$ BHA por el teorema de Pitágoras:

$a^2=6^2+8^2$

$\Rightarrow a^2=100$

$\Rightarrow a=10$

Piden: el perímetro (2p) del $\triangle$ABC.

$2p=AB+BC+AC$

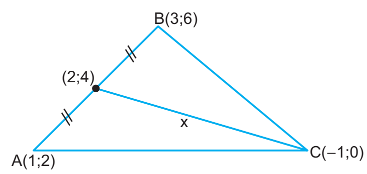
$2p=a+a+16=2a+16$

$\Rightarrow 2p=2(10)+16=36$

$\therefore\ 2p=36$

Clave B

24.



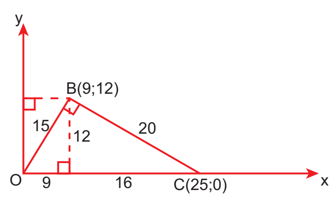
$(2-(-1))^2+(4-0)^2=x^2$

$9+16=x^2$

$\Rightarrow x=5$

Clave D

25.



En el $\triangle$OHB por el teorema de Pitágoras:

$OB=15$

En el $\triangle$BHC por el teorema de Pitágoras:

$BC=20$

Luego, el triángulo OBC cumple con el teorema de Pitágoras, es decir:

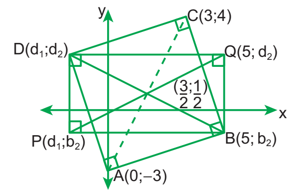
$(OC)^2=(OB)^2+(BC)^2$

$\Rightarrow m\angle OBC=90°$; además, sus lados tienen diferentes medidas.

Por lo tanto, el triángulo OBC es rectángulo y escaleno.

Clave E

26.



Del gráfico:

$$\frac{d_1+5}{2}=\frac{3}{2}\Rightarrow d_1+5=3$$

$$\Rightarrow d_1=-2 \qquad ...(1)$$

$$\frac{d_2+b_2}{2}=\frac{1}{2}\Rightarrow d_2+b_2=1$$

$$(d_1-5)^2+(d_2-b_2)^2=(3-0)^2+(4-(-3))^2$$
$$=7^2+3^2=58$$

Reemplazando $d_1=-2$ se tiene:

$(-2-5)^2+(d_2-b_2)^2=58$

$(-7)^2+(d_2-b_2)^2=58$

$\Rightarrow (d_2-b_2)^2=9;\ d_2-b_2>0$

$\Rightarrow d_2-b_2=3 \qquad ...(2)$

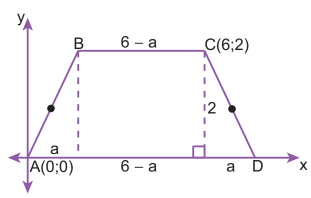
$A=(5-d_1)(d_2-b_2) \quad ...(3)$

Reemplazando (1) y (2) en (3):

$\therefore A=(5-(-2))(d_2-b_2)=7.3=21$

Clave D

27.

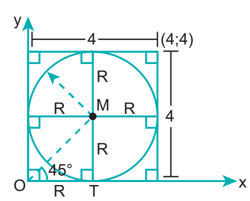


$$A=\left(\frac{6+a+6-a}{2}\right)2$$

$\therefore A=12$

Clave A

28.



Del gráfico: $2R=4\Rightarrow R=2$

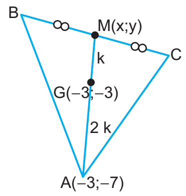
Del $\triangle$OTM notable de 45°:

$\Rightarrow OM=R\sqrt{2}=(2)\sqrt{2}$

$\therefore OM=2\sqrt{2}$

Clave C

29.



Por dato: G es baricentro del $\triangle ABC$.
Del gráfico: G(–3; –3) divide al segmento AM en la razón $r = \dfrac{AG}{GM} = \dfrac{2k}{k} \Rightarrow r = 2$

$$\Rightarrow -3 = \frac{(-3)+2(x)}{1+2} \Rightarrow x = -3$$

$$\Rightarrow -3 = \frac{(-7)+2(y)}{1+2} \Rightarrow y = -1$$

$\therefore M(x; y) = M(-3; -1)$

Clave D

Resolución de problemas

30. • Hallamos las coordenadas del baricentro.

$$G(x; y) = \frac{A(x_1; y_1) + B(x_2; y_2) + C(x_3; y_3)}{3}$$

$$G(x; y) = \frac{(-3; 3) + (-3; -4) + (3; -2)}{3}$$

$$G(x; y) = \frac{(-3; -3)}{3}$$

$G(x; y) = (-1; -1)$

• Ahora podemos obtener el punto medio de $\overline{AG}$:

$$M(a; b) = \frac{A+G}{2}$$

$$M(a; b) = \frac{(-3; 3) + (-1; -1)}{2}$$

$$M(a; b) = \frac{(-4; 2)}{2} = (-2; 1)$$

$\therefore\ a = -2 \ \wedge\ b = 1 \Rightarrow\ a + b = -1$

Clave C

31. • El área sombreada lo obtenemos de la siguiente forma:

$A_S = A_{cuadrado} - A_{círculo}$

$A_S = (lado)^2 - \pi(radio)^2$

$A_S = (AB)^2 - \pi\left(\dfrac{AB}{2}\right)^2$

$A_S = (AB)^2(1 - \pi/4)$ ... (α)

• $M(1; 2) = \dfrac{A(x; y) + O(o; o)}{2}$

$2M(1; 2) = A(x; y) \Rightarrow\ A(x; y) = (2; 4)$

• Hallamos la distancia AB:

$AB = \sqrt{(2-4)^2 + (4-2)^2}$

$(AB)^2 = (-2)^2 + (2)^2$

$(AB)^2 = 4 + 4 = 8$

• Reemplazamos en (α):

$A_S = (AB)^2\left(1 - \dfrac{\pi}{4}\right)$

$A_S = 8\left(1 - \dfrac{\pi}{4}\right)$

$\therefore A_S = 8 - 2\pi$

Clave A

MARATÓN MATEMÁTICA (página 50)

1.

$$B = -1 + \cfrac{1}{1 - \cfrac{1}{1 + \cfrac{sen^2x}{(1 - senx)(1 + senx)}}}$$

$$B = -1 + \cfrac{1}{1 - \cfrac{1}{1 + \cfrac{sen^2x}{1 - sen^2x}}}$$

$$B = -1 + \cfrac{1}{1 - \cfrac{1}{sec^2x}}$$

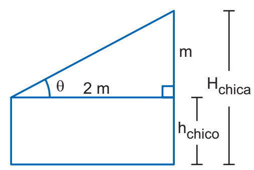
$$B = -1 + \frac{1}{1 - cos^2x} = -1 + \frac{1}{sen^2x}$$

$B = -1 + csc^2x = csc^2x - 1$

$\therefore\ B = cot^2x$

Clave A

2.



Del gráfico: $tan\theta = 1/2$

Nos piden: $M = (sec\theta - 1)(sec\theta + 1)$

$M = sec^2\theta - 1$

$M = tan^2\theta$

$M = (1/2)^2$

$\therefore\ M = 1/4$

Clave C

3. $R = cos(tan(sen\pi)) + tan\left(cos\left(cos\left(\dfrac{3\pi}{2}\right)\right)\right)$

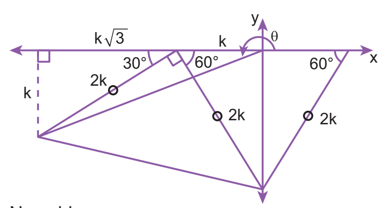
$R = cos(tan(0)) + tan(cos(0))$

$R = cos(0) + tan(1)$

$R = 1 + tan1$

Clave A

4.



Nos piden:

$P = cot^2\theta - 1 = \left(\dfrac{(-\sqrt{3} + 1)}{-1}\right)^2 - 1$

$P = 3 + 1 + 2\sqrt{3} - 1$

$P = 3 + 2\sqrt{3}$

Clave B

5. $2^{cotx - 2} = (\sqrt{2})^{cotx}$

$2^{cotx - 2} = 2^{\frac{cotx}{2}}$

$\Rightarrow\ 2cotx - 2 = \dfrac{cotx}{2}$

$\dfrac{3\,cotx}{2} = 2 \Rightarrow cotx = \dfrac{4}{3}$

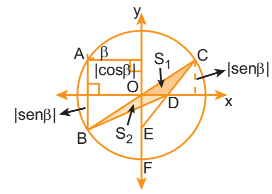
• Piden ($x \in IC$):

$A = 2senx + cosx$

$A = 2\left(\dfrac{3}{5}\right) + \dfrac{4}{5} = \dfrac{10}{5} = 2$

Clave E

6. En el gráfico:



$S = S_1 + S_2$

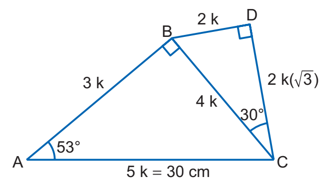
$S = \dfrac{1}{2} \times \dfrac{|cos\beta|}{2} \times |sen\beta| + \dfrac{1}{2} \times \dfrac{|cos\beta|}{2} \times |sen\beta|$

$S = \dfrac{1}{4}(-cos\beta sen\beta) + \dfrac{1}{4}(-cos\beta sen\beta)$

$S = \dfrac{1}{4}(-2sen\beta cos\beta) = \dfrac{-sen\beta\, .\, cos\beta}{2}$

Clave B

7. • Del gráfico tenemos:



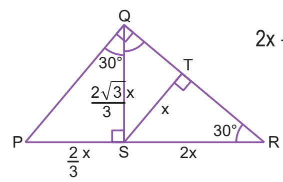
• $A_{\triangle BDC} = \dfrac{2k}{2}(2k\sqrt{3}) = 2\sqrt{3}\,k^2$

$A_{\triangle BDC} = 2\sqrt{3}\left(\dfrac{30\ cm}{5}\right)^2$

$\therefore\ A_{\triangle BDC} = 72\sqrt{3}\ cm^2$

Clave D

8.



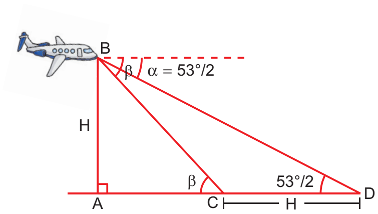
$2x + \dfrac{2x}{3} = 48\ cm$

$\dfrac{8x}{3} = 48\ cm$

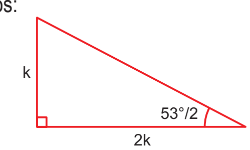
$\therefore\ x = 18\ cm$

Clave B

9.



Sabemos:



Entonces:

$AD = 2H$

$AC + CD = 2H$

$AC = H$

$tan\beta = \dfrac{H}{H} = 1$

$\therefore\ \beta = 45°$

Clave C

RAZONES TRIGONOMÉTRICAS DE ÁNGULOS EN CUALQUIER MAGNITUD

APLICAMOS LO APRENDIDO (página 53) Unidad 3

1. $(-5; 12) = (x; y) \Rightarrow x = -5 \wedge y = 12$

$x^2 + y^2 = r^2 \Rightarrow (-5)^2 + (12)^2 = r^2$

$25 + 144 = r^2 \Rightarrow r^2 = 169 \Rightarrow r = 13$

Reemplazamos en M:

$$M = \frac{\sec\alpha + \tan\alpha}{\operatorname{sen}\alpha + \cos\alpha} = \frac{\frac{r}{x} + \frac{y}{x}}{\frac{y}{r} + \frac{x}{r}} = \frac{\frac{r+y}{x}}{\frac{y+x}{r}}$$

$$M = = \frac{\frac{(13+12)}{-5}}{\frac{12-5}{13}} \Rightarrow M = \frac{\left(\frac{25}{-5}\right)}{\left(\frac{7}{13}\right)} = \frac{-65}{7}$$

Clave D

2. Hallamos las coordenadas de M (punto medio):

$M(x; y) = \frac{A+B}{2} = \left(\frac{-6+0}{2}; \frac{8+0}{2}\right)$

$\Rightarrow x = -3 \wedge y = 4$

Además:

$x^2 + y^2 = r^2$

$\Rightarrow (-3)^2 + (4)^2 = r^2$

$r = 5$

Reemplazamos en K:

$$K = \frac{\operatorname{sen}\alpha + \cos\alpha}{\tan\alpha} = \frac{\frac{y}{r} + \frac{x}{r}}{\frac{y}{x}} = \frac{\frac{y+x}{r}}{\frac{y}{x}}$$

$$= \frac{\frac{-3+4}{5}}{\frac{4}{-3}} = \frac{-3}{20}$$

Clave A

3. En la sumatoria:

$\theta = 1° + 2° + 3° + ... + 26° = \frac{26°(27°)}{2} = 351°$

$\therefore \theta = 351°$

$\theta \in \text{IVC} \Rightarrow \operatorname{sen}\theta\ (-) \wedge \tan\theta\ (-)$

Clave C

4. Sean los ángulos α y β; además $\alpha < \beta$.

$\frac{\alpha}{\beta} = \frac{1}{7} = k \Rightarrow \begin{matrix}\alpha = k \\ \beta = 7k\end{matrix}$

Como son coterminales se cumple:

$\beta - \alpha = 360°\ n;\ n \in \mathbb{Z}^+ - \{0\}$

$7k - k = 360°.\ n \Rightarrow 6k = 360°.\ n \Rightarrow k = 60°.\ n$

Reemplazamos:

Si $n = 1 \Rightarrow k = 60° \Rightarrow \alpha = 60° \wedge \beta = 420°$

Si $n = 2 \Rightarrow k = 120° \Rightarrow \alpha = 120° \wedge \beta = 840°$

Si $n = 3 \Rightarrow k = 180° \Rightarrow \alpha = 180° \wedge \beta = 1260°$

Si $n = 4 \Rightarrow k = 240° \Rightarrow \alpha = 240° \wedge \beta = 1640°$

Hallamos la suma:

$\alpha + \beta = 180° + 1260° = 1440°$

Clave A

5. Si (x; y) un punto del lado final de β.

$x^2 + y^2 = r^2 \wedge \cot\beta = \frac{x}{y} = \frac{1}{2} \Rightarrow \begin{matrix}x = \pm 1 \\ y = \pm 2\end{matrix}$

$\beta \in \text{IIIC} \Rightarrow \begin{matrix}x = -1 \\ y = -2\end{matrix} \Rightarrow (-1)^2 + (-2)^2 = r^2$

$1 + 4 = r^2 \Rightarrow r = \sqrt{5}$

Hallamos el valor de R:

$$R = \frac{1 + \cos\beta}{1 - \cos\beta} = \frac{1 + \frac{x}{r}}{1 - \frac{x}{r}} = \frac{\frac{r+x}{r}}{\frac{r-x}{r}}$$

$$R = \frac{r+x}{r-x} = \frac{\sqrt{5} + (-1)}{\sqrt{5} - (-1)}$$

$$R = \frac{\sqrt{5} - 1}{\sqrt{5} + 1} \times \frac{\sqrt{5} - 1}{\sqrt{5} - 1}$$

$$R = \frac{5 - 2\sqrt{5} + 1}{4} = \frac{6 - 2\sqrt{5}}{4}$$

$$R = \frac{(3 - \sqrt{5})}{2}$$

Clave C

6. Hallamos M:

$M = \frac{A+B}{2} = \left(\frac{-2a + (-2a)}{2}; \frac{2a+0}{2}\right)$

$M = (-2a; a)$

Hallamos las coordenadas de E:

$E = \frac{M+D}{2} = \left(\frac{-2a+0}{2}; \frac{a+2a}{2}\right)$

$E = \left(-a; \frac{3}{2}a\right)$

Reemplazamos en K:

$$K = \cot\alpha - \tan\alpha = \frac{x}{y} - \frac{y}{x} = \left(\frac{-a}{\frac{3}{2}a}\right) - \left(\frac{\frac{3}{2}a}{-a}\right)$$

$$K = \frac{-2}{3} + \frac{3}{2} = \frac{5}{6}$$

Clave B

7. $\theta \in\ ]80°; 100°]$

$80° < \theta \le 100° \Rightarrow 40° < \frac{\theta}{2} \le 50° \Rightarrow \frac{\theta}{2} \in \text{IC}$

$80° < \theta \le 100° \Rightarrow 20° < \frac{\theta}{4} \le 25° \Rightarrow \frac{\theta}{4} \in \text{IC}$

$80° < \theta \le 100° \Rightarrow 120° < \frac{3\theta}{2} < 150° \Rightarrow \frac{3\theta}{2} \in \text{IIC}$

Piden el signo de:

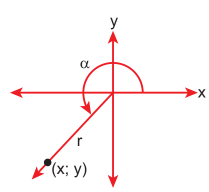
$P = \tan\frac{\theta}{2}\ .\ \cos\frac{\theta}{4} = (+)\ .\ (+) = (+)$

$J = \sec\frac{3\theta}{2} - \csc\theta = (-) - (+) = (-)$

Por lo tanto, los signos serán: (+); (–)

Clave B

8.



Por dato: $\tan\alpha = \frac{1}{3}$

$\frac{y}{x} = \frac{1k}{3k} \Rightarrow y = k \wedge x = 3k$

Por radio vector: $r = \sqrt{10}\,k$

Piden: $P = 3\sec\alpha - \csc\alpha = 3\left(\frac{r}{x}\right) - \left(\frac{r}{y}\right)$

$P = 3\left(\frac{\sqrt{10}\,k}{3k}\right) - \left(\frac{\sqrt{10}\,k}{k}\right) = \sqrt{10} - \sqrt{10} = 0$

$\therefore P = 0$

Clave A

9. $\sqrt{\cos\alpha + 1} + \sqrt{-1 - \cos\alpha} = 1 - \operatorname{sen}\theta$

Se debe de cumplir:

$\cos\alpha + 1 \ge 0 \wedge -1 - \cos\alpha \ge 0$

$\cos\alpha \ge -1 \wedge -1 \ge \cos\alpha$

De ambas condiciones se deduce que:

$\cos\alpha = -1$

$\Rightarrow \alpha = 180°$

Reemplazando:

$\sqrt{(-1) + 1} + \sqrt{-1 - (-1)} = 1 - \operatorname{sen}\theta$

$0 + 0 = 1 - \operatorname{sen}\theta$

$\operatorname{sen}\theta = 1$

$\Rightarrow \theta = 90°$

$\therefore \alpha = 180° \wedge \theta = 90°$

Clave B

10. $1 - \cos^2\theta = \frac{1}{4} \Rightarrow \operatorname{sen}^2\theta = \frac{1}{4}$

$\Rightarrow \operatorname{sen}\theta = \pm\frac{1}{2}; \theta \in \text{IIIC} \Rightarrow \operatorname{sen}\theta = -1/2$

$y = -1 \wedge r = 2$

$x^2 + y^2 = r^2 \Rightarrow x^2 + (-1)^2 = (2)^2$

$x^2 + 1 = 4$

$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

$\theta \in \text{IIIC} \Rightarrow x = -\sqrt{3}$

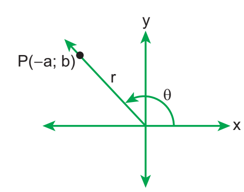
Hallamos el valor de A:

$A = \sec^2\theta + 1 = \left(\frac{r}{x}\right)^2 + 1$

$A = \left(\frac{2}{\sqrt{3}}\right)^2 + 1 = \frac{4}{3} + 1 \Rightarrow A = 7/3$

Clave E

11.



Por radio vector: $r = \sqrt{a^2 + b^2}$

$\cos\theta = \frac{-a}{\sqrt{a^2 + b^2}}$

$\cot\theta = \frac{-a}{b}$

Piden: $A = \sqrt{\sqrt{a^2 + b^2}\ .\cos\theta.\cot\theta.b}$

$$A = \sqrt{\sqrt{a^2 + b^2}\ .\frac{-a}{\sqrt{a^2 + b^2}}.\frac{-a}{b}.b}$$

$A = \sqrt{a^2} = |a|$; como $-a < 0 \Rightarrow a > 0$

$\therefore A = a$

Clave B

12. $M = \dfrac{\text{sen}720° + \cos 2160°}{\text{sen}1530° - \tan 1440°}$

Notamos que los ángulos son múltiplos de 90°, entonces coincidirán en valor con los correspondientes ángulos cuadrantales conocidos, para ello calculamos cuántas vueltas enteras contiene cada uno para saber su ubicación.

$$\begin{array}{r|l} 720° & 360° \\ \hline 0° & 2 \end{array} \qquad \begin{array}{r|l} 2160° & 360° \\ \hline 0° & 6 \end{array}$$

$$\begin{array}{r|l} 1530° & 360° \\ \hline 90° & 4 \end{array} \qquad \begin{array}{r|l} 1440° & 360° \\ \hline 0° & 4 \end{array}$$

Entonces la expresión equivalente a M será:

$M = \dfrac{\text{sen}360° + \cos 360°}{\text{sen}90° - \tan 360°}$

$M = \dfrac{0+1}{1-0} = \dfrac{1}{1} = 1 \qquad \therefore M = 1$

Clave D

13. $\alpha \in \text{IIC} \Rightarrow 90 < \alpha < 180°$

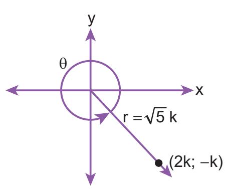
$45 < \alpha/2 < 90 \Rightarrow \alpha/2 \in \text{IC}$

$M = \dfrac{\text{sen}\alpha \cos\beta \tan\alpha}{\csc\alpha + \cot\beta} = \dfrac{(+).(-)(-)}{(+).+(+)} = \dfrac{(+)}{(+)} = (+)$

$N = \dfrac{\tan\beta - \text{sen}\beta}{\csc\left(\frac{\alpha}{2}\right)\text{sen}\left(\frac{\alpha}{2}\right)} = \dfrac{(+)-(-)}{(+)(+)} = \dfrac{(+)(+)}{(+)(+)} = (+)$

Clave C

14.



$\theta \in \text{IV} \Rightarrow \tan\theta < 0$

Por dato: $\tan^2\theta = \dfrac{1}{4}$

$\Rightarrow \tan\theta = \pm\dfrac{1}{2} \Rightarrow \tan\theta = \dfrac{-1}{2}$

Piden:

$R = 2\sec\theta + \csc\theta$

$R = 2\left(\dfrac{r}{x}\right) + \left(\dfrac{r}{y}\right) = 2\left(\dfrac{\sqrt{5}\,k}{2k}\right) + \left(\dfrac{\sqrt{5}\,k}{-k}\right)$

$R = \sqrt{5} - \sqrt{5} = 0$

$\therefore R = 0$

Clave D

PRACTIQUEMOS

Nivel 1 (página 55) Unidad 3

Comunicación matemática

1.

R. T. / m<	sen+cos	sen–cos	sec+1	csc–1	cos+sec
0°	1	–1	2	N D	2
90°	–1	1	N D	0	N D
180°	–1	1	1	N D	–2
270°	1	–1	N D	–2	ND

2. Si $\theta \in \text{IIIC} \Rightarrow \text{sen}\theta + \cos\theta = (-) + (-) = (-)$

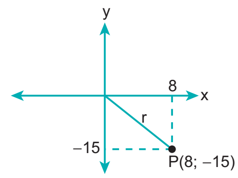
Si $\theta \in \text{IVC} \Rightarrow \cos\theta - \tan\theta = (+) - (-) = (+)$

Si $\theta \in \text{IIC} \Rightarrow \text{sen}\theta\cos\theta = (+)(-) = (-)$

Si $\theta \in \text{IC} \Rightarrow (\text{sen}\theta - 1)(\text{sen}\theta + 1) = (-)(+) = (-)$

Razonamiento y demostración

3.



$x^2 + y^2 = r^2 \Rightarrow (8)^2 + (-15)^2 = r^2$

$\Rightarrow r = 17$

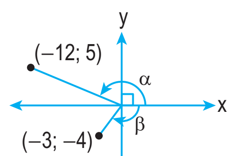
$L = 2\text{sen}\beta - \dfrac{1}{2}\cos\beta$

$L = 2\left(\dfrac{-15}{17}\right) - \dfrac{1}{2}\left(\dfrac{8}{17}\right)$

$L = -\dfrac{30}{17} - \dfrac{4}{17} = -\dfrac{34}{17} = -2$

Clave D

4.



$r_1^2 = (-12)^2 + (5)^2 \Rightarrow r_1 = 13$

$r_2^2 = (-3)^2 + (-4)^2 \Rightarrow r_2 = 5$

Entonces:

$\csc\alpha = \dfrac{13}{5} \qquad \cos\beta = -\dfrac{3}{5}$

$\therefore E = \csc\alpha + \cos\beta = \dfrac{13}{5} - \dfrac{3}{5} = 2$

Clave B

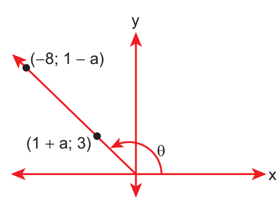
5. $\text{sen}\theta < 0 \quad \wedge \quad \tan\theta > 0$

$(\text{IIIC} \vee \text{IVC}) \quad \wedge \quad (\text{IC} \vee \text{IIIC})$

$\Rightarrow \theta \in \text{IIIC}$

Clave C

6.



$\tan\theta = \dfrac{y}{x} = \dfrac{1-a}{-8} = \dfrac{3}{1+a}$

$\Rightarrow 1 - a^2 = -24$

$25 = a^2 \Rightarrow a = -5$

$\tan\theta = \dfrac{y}{x} = \dfrac{1-(-5)}{-8} = -\dfrac{6}{8} = -\dfrac{3}{4}$

$\Rightarrow a - 8\tan\theta = -5 - 8 \cdot \left(\dfrac{-3}{4}\right)$

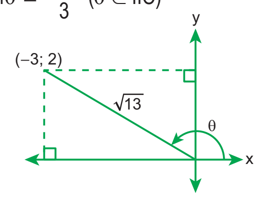
$= -5 + 6 = 1$

Clave E

7. $3\tan\theta + 2 = \cos 90° = 0$

$3\tan\theta = -2$

$\Rightarrow \tan\theta = -\dfrac{2}{3} \quad (\theta \in \text{IIC})$

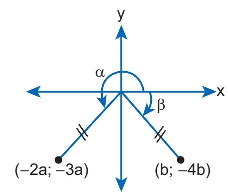


$\Rightarrow E = \text{sen}\theta + \cos\theta = \dfrac{2}{\sqrt{13}} - \dfrac{3}{\sqrt{13}}$

$\therefore E = -\dfrac{1}{\sqrt{13}} \cdot \dfrac{\sqrt{13}}{\sqrt{13}} = -\dfrac{\sqrt{13}}{13}$

Clave D

8.



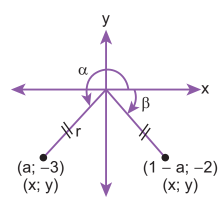
$\tan\alpha = \dfrac{y}{x} = \dfrac{-3a}{-2a} = \dfrac{3}{2}$

$\cot\beta = \dfrac{x}{y} = \dfrac{b}{-4b} = -\dfrac{1}{4}$

$\therefore \tan\alpha\cot\beta = \left(\dfrac{3}{2}\right)\left(-\dfrac{1}{4}\right) = -\dfrac{3}{8}$

Clave D

9.



$\tan\alpha = \dfrac{y}{x} = \dfrac{-3}{a} \qquad \tan\beta = \dfrac{-2}{1-a}$

$r^2 = a^2 + (-3)^2 = (1-a)^2 + (-2)^2$

$a^2 + 9 = a^2 - 2a + 1 + 4$

$2a = -4$

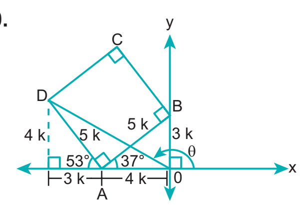
$\Rightarrow a = -2$

$2\tan\alpha + 3\tan\beta = 2\left(\dfrac{-3}{-2}\right) + 3\left(\dfrac{-2}{3}\right) = 3 - 2 = 1$

Clave A

Resolución de problemas

10.



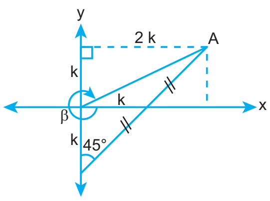
El punto D:
$(-7k; 4k)$

Hallamos la tangente de θ:

$\tan\theta = \dfrac{y}{x} = \dfrac{4k}{-7k} = -\dfrac{4}{7}$

Clave A

11.



El punto A tiene las siguientes coordenadas:
$A(x; y) = (2k; k)$

Hallamos el valor de R:

$R = \tan\beta + \cot\beta = \dfrac{y}{x} + \dfrac{x}{y}$

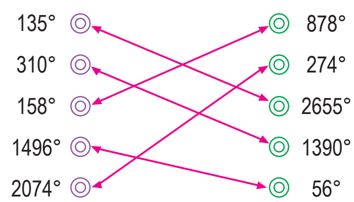
$R = \dfrac{1}{2} + \dfrac{2}{1} = \dfrac{5}{2}$

Clave E

Nivel 2 (página 55) Unidad 3

Comunicación matemática

12. Tenemos en cuenta: $\alpha = \beta + n(360°)$



13. I. $\text{sen}1134°.\cos148° < 0$
$(+)\ .\ (-)\ < 0$ (V)

II. $\tan576°\ .\ \sec220° > 0$
$(+)\ .\ (-)\ > 0$ (F)

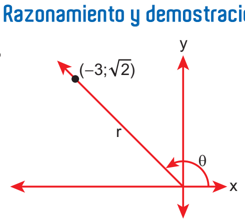
III. $2\text{sen}90° + 2\sec180° = 0$
$2(1) + 2(-1) = 0$ (V)

IV. $3\text{sen}270° + 4\sec360° < 0$
$3(-1) + 4(1) < 0$ (F)

Clave D

Razonamiento y demostración

14.



$r^2 = x^2 + y^2$

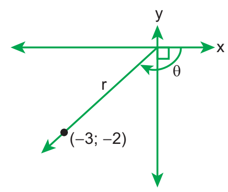
$r^2 = (-3)^2 + (\sqrt{2})^2$

$\Rightarrow r = \sqrt{11}$

$\cos\theta = \dfrac{x}{r} = \dfrac{-3}{\sqrt{11}}$

Clave D

15.



$r^2 = (-3)^2 + (-2)^2$

$\Rightarrow r = \sqrt{13}$

$\therefore\ \text{sen}\theta = -\dfrac{2}{\sqrt{13}}$

Clave D

16. $8^{\tan\theta + 1} = 4$

$2^{3(\tan\theta + 1)} = 2^2$

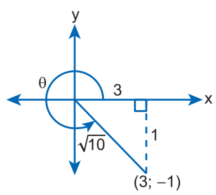
$\Rightarrow 3(\tan\theta + 1) = 2$

$\tan\theta + 1 = \dfrac{2}{3} \Rightarrow \tan\theta = -\dfrac{1}{3}$ ...(1)

Como:
$\cos\theta > 0$ ...(2)

De (1) y (2):
$\Rightarrow \theta \in \text{IVC}$

$\tan\theta = \dfrac{-1}{3} = \dfrac{y}{x}$



$\therefore\ \text{sen}\theta = \dfrac{y}{r} = \dfrac{-1}{\sqrt{10}}$

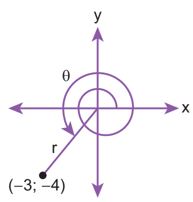
Clave B

17. $\text{sen}\alpha > 0,\ \cos\alpha < 0$
$\Rightarrow \alpha \in \text{IIC}$
$\Rightarrow \tan\alpha\ ...\ (-)\ \wedge\ \cot\alpha\ ...\ (-)$

$(\tan\alpha + \cot\alpha)\text{sen}\alpha$
$((-) + (-))(+) = (-)(+) = (-)$

Clave B

18.



Por radio vector: $r^2 = x^2 + y^2$

$\Rightarrow r^2 = (-3)^2 + (-4)^2 = 25$

$\Rightarrow r = 5$

Piden:

$1 - \text{sen}\theta = 1 - \left(\dfrac{y}{r}\right) = 1 - \left(\dfrac{-4}{5}\right)$

$\Rightarrow 1 - \text{sen}\theta = 1 + \dfrac{4}{5} = \dfrac{9}{5} = 1{,}8$

$\therefore 1 - \text{sen}\theta = 1{,}8$

Clave E

19. Por dato: $\csc^2\theta = 4$
$\Rightarrow \csc\theta = 2 \vee \csc\theta = -2$

Además: $\theta \in \text{IIIC} \Rightarrow \csc\theta < 0$
$\Rightarrow \csc\theta = -2$

Luego:

$\csc\theta = \dfrac{r}{y} = \dfrac{2}{-1} \Rightarrow r = 2 \wedge y = -1$

Por radio vector: $x^2 + y^2 = r^2$
$\Rightarrow x^2 + (-1)^2 = 2^2 \Rightarrow x^2 = 3$
$\Rightarrow x = \sqrt{3} \vee x = -\sqrt{3}$

Como $\theta \in \text{IIIC} \Rightarrow x < 0$
$\Rightarrow x = -\sqrt{3}$

Piden:

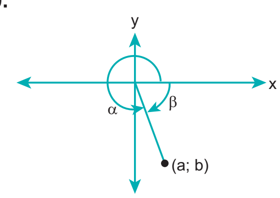
$M = \dfrac{\csc\theta}{\sec\theta + 2\cot\theta} = \dfrac{\left(\dfrac{r}{y}\right)}{\left(\dfrac{r}{x}\right) + 2\left(\dfrac{x}{y}\right)}$

$M = \dfrac{\left(\dfrac{2}{-1}\right)}{\left(\dfrac{2}{-\sqrt{3}}\right) + 2\left(\dfrac{-\sqrt{3}}{-1}\right)} = \dfrac{-2}{\left(\dfrac{4}{\sqrt{3}}\right)}$

$\therefore\ M = -\dfrac{\sqrt{3}}{2}$

Clave C

20.



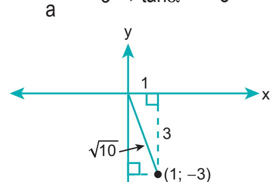
$\tan\alpha = \dfrac{b}{a}$

$\tan\beta = \dfrac{b}{a}$

$\Rightarrow \tan\alpha + \tan\beta = -6$

$\dfrac{b}{a} + \dfrac{b}{a} = -6$

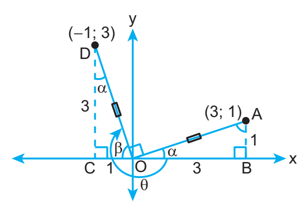
$\dfrac{b}{a} = -3 \Rightarrow \tan\alpha = -3$



$\Rightarrow \text{sen}\alpha = \dfrac{-3}{\sqrt{10}} = \dfrac{-3\sqrt{10}}{10}$

Clave D

21.



Hacemos $\overline{OD} = \overline{OA} \Rightarrow \triangle OCD \cong \triangle OBA$
$\Rightarrow D(-1; 3)$

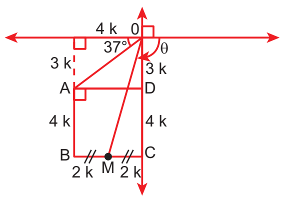
Piden: $\tan\theta = \dfrac{y}{x} = \dfrac{3}{-1} = -3$

$\therefore \tan\theta = -3$

Clave D

Resolución de problemas

22.



Las coordenadas de M:

$M = (-2k; -7k)$

Hallamos la $\tan\theta$:

$\tan\theta = \dfrac{y}{x} = \dfrac{-7k}{-2k} = \dfrac{7}{2}$

Clave B

23. Sean α y β los ángulos y $\alpha > \beta$.

$\dfrac{\alpha}{\beta} = \dfrac{11}{2} = k \Rightarrow \begin{matrix} \alpha = 11k \\ \beta = 2k \end{matrix}$

$\alpha - \beta = 360\,n$

$11k - 2k = 360\,n \Rightarrow k = 40\,n$

$\dfrac{\alpha}{10} + \dfrac{\beta}{5} = 180°$

$\dfrac{11k}{10} + \dfrac{2k}{5} = 180° \Rightarrow \dfrac{11k + 4k}{10} = 180°$

$\dfrac{15k}{10} = 180°$

$k = 120°$

$\alpha + \beta = 11k + 2k = 13k$

$\alpha + \beta = 1560°$

Clave A

Nivel 3 (página 56) Unidad 3

Comunicación matemática

24. $M = [2\text{sen}\frac{\pi}{2} - 3\cos\pi]^2$

$M = [2\text{sen}90° - 3\cos180°]^2$

$M = [2(1) - 3(-1)]^2 = [5]^2 = 25$

$N = 4\text{sen}^3\frac{3\pi}{2} + 3\sec^2\pi + (2\sec2\pi)^4$

$N = 4\text{sen}^3 270° + 3\sec^2 180° + (2\sec360°)^4$

$N = 4(-1)^3 + 3(-1)^2 + (2(1))^4$

$N = -4 + 3 + 16 = 15$

Clave B

25. I. Si $\theta \in$ IIC $\Rightarrow$ $\text{sen}\theta\tan^3\theta > 0$

$(+) \,.\, (-)^3 > 0$ (F)

II. Si $\theta \in$ IIIC $\Rightarrow$ $\cos\theta\cot\theta + \text{sen}\theta < 0$

$(-) \,.\, (+) + (-) < 0$

$(-) + (-) < 0$ (V)

III. Si $\theta \in$ IIC $\Rightarrow$ $(-\theta) \in$ IIIC

$\cos(-\theta)\tan(-\theta) > 0$

$(-) \,.\, (+) > 0$ (F)

IV. Si $\theta \in$ IVC $\Rightarrow$ $(-\theta) \in$ IC

$\text{sen}(-\theta)\sec(-\theta) > 0$

$(+) \,.\, (+) > 0$ (V)

Clave C

Razonamiento y demostración

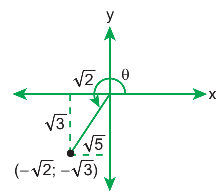
26. $4^{\tan^2\theta} = 8 \qquad \theta \in$ IIIC

$2^{2\tan^2\theta} = 2^3 \Rightarrow \tan\theta \ldots (+)$

$2\tan^2\theta = 3 \Rightarrow \text{sen}\theta \ldots (-)$

$\tan^2\theta = \dfrac{3}{2} \Rightarrow \cos\theta \ldots (-)$

$\Rightarrow \tan\theta = \dfrac{\sqrt{3}}{\sqrt{2}}$



$\text{sen}\theta = -\dfrac{\sqrt{3}}{\sqrt{5}} \quad \wedge \quad \cos\theta = -\dfrac{\sqrt{2}}{\sqrt{5}}$

$E = \sqrt{3}\,\text{sen}\theta + \sqrt{2}\cos\theta$

$E = \sqrt{3}\left(\dfrac{-\sqrt{3}}{\sqrt{5}}\right) + \sqrt{2}\left(\dfrac{-\sqrt{2}}{\sqrt{5}}\right) = \dfrac{-5(\sqrt{5})}{\sqrt{5}(\sqrt{5})}$

$\therefore E = -\sqrt{5}$

Clave B

27. $\tan\theta - 2 = \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)}}}$

$\tan\theta - 2 = \cfrac{1}{4 + \cfrac{1}{\sqrt{5}+2}}$

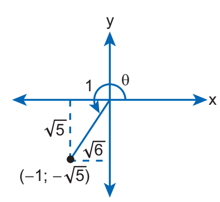
$\tan\theta - 2 = \cfrac{1}{4 + \cfrac{1(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)}}$

$\tan\theta - 2 = \dfrac{1}{4 + \sqrt{5} - 2} = \dfrac{1}{(\sqrt{5}+2)}\dfrac{(\sqrt{5}-2)}{(\sqrt{5}-2)}$

$\tan\theta - 2 = \sqrt{5} - 2$

$\tan\theta = \sqrt{5} = \dfrac{\sqrt{5}}{1} = \dfrac{y}{x}$

Además: $\theta \in$ IIIC



$\therefore \sqrt{5}\csc\theta = \sqrt{5}\left(\dfrac{\sqrt{6}}{-\sqrt{5}}\right) = -\sqrt{6}$

Clave E

28.

$\overbrace{\text{sen}\alpha}^{(-)}\underbrace{\sqrt{\cos\alpha}}_{(+)} < 0 \Rightarrow \alpha \in$ IVC

$\Rightarrow P = \dfrac{\cos\alpha}{\text{sen}\alpha + \tan\alpha} = \dfrac{(+)}{(-)+(-)} = \dfrac{(+)}{(-)} = (-)$

Clave B

29. $\sqrt{\tan\alpha^{\cos\beta-1}} = \cot\alpha^{\frac{2}{3}} = (\tan\alpha)^{-\frac{2}{3}}$

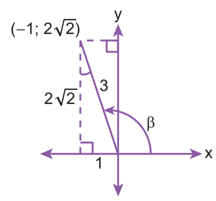
$\tan\alpha^{\frac{\cos\beta-1}{2}} = \tan\alpha^{\frac{-2}{3}}$

$\dfrac{(\cos\beta-1)}{2} = -\dfrac{2}{3}$

$\dfrac{1-\cos\beta}{2} = \dfrac{2}{3} \Rightarrow 3 - 3\cos\beta = 4$
$\qquad\qquad\qquad\qquad -3\cos\beta = 1$

$\Rightarrow \cos\beta = \dfrac{-1}{3}$

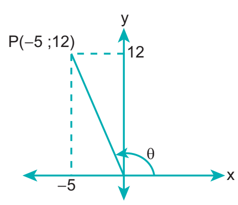
Como: $\beta \notin \text{IIIC} \Rightarrow \beta \in \text{IIC}$



$\Rightarrow \tan\beta = -2\sqrt{2} \ \wedge\ \sec\beta = -3$
$\therefore C = \sqrt{2}(-2\sqrt{2}) - 3 = -4 - 3 = -7$

Clave A

30.



$r^2 = 12^2 + 5^2$
$\Rightarrow r = 13$

$\text{sen}\theta = \dfrac{y}{r} = \dfrac{12}{13}$

$\cos\theta = \dfrac{x}{r} = \dfrac{-5}{13}$

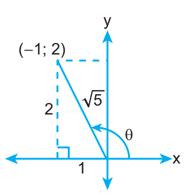
$L = 5\text{sen}\theta - \cos\theta = \dfrac{5(12)}{13} - \dfrac{(-5)}{13}$

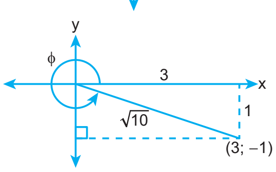
$\therefore L = \dfrac{65}{13} = 5$

Clave E

31. Dato:
$\theta \in \text{IIC y } \phi \in \text{IVC}$ ...(1)
$\tan\theta + 2 = 0 \ \wedge\ \cot\phi + 3 = 0$
$\tan\theta = -2 \qquad \cot\phi = -3$ ...(2)

De (1) y (2):

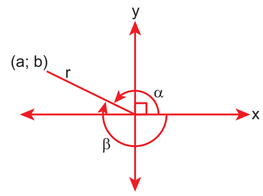




$\therefore \sqrt{2}\cos\theta\cos\phi = \sqrt{2}\left(\dfrac{-1}{\sqrt{5}}\right)\left(\dfrac{3}{\sqrt{10}}\right) = -\dfrac{3}{5}$

Clave C

32.



$\tan\alpha = \dfrac{b}{a}$

$\tan\beta = \dfrac{b}{a}$

$\cos\alpha = \dfrac{a}{r}$

$\cos\beta = \dfrac{a}{r}$

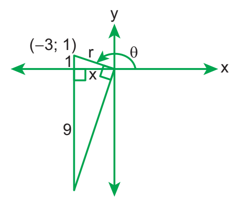
$\Rightarrow E = \dfrac{\tan\alpha}{\tan\beta} + \cos\alpha - \cos\beta$

$\therefore E = \dfrac{\frac{b}{a}}{\frac{b}{a}} + \dfrac{a}{r} - \dfrac{a}{r} = 1$

Clave A

Resolución de problemas

33.



$x^2 = 1(9)$
$x^2 = 9$
$\Rightarrow x = 3$

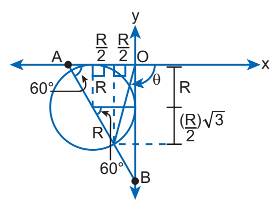
$r^2 = (-3)^2 + (1)^2$

$\Rightarrow r = \sqrt{10}$

$\therefore \text{sen}\theta = \dfrac{1}{\sqrt{10}} = \dfrac{\sqrt{10}}{10}$

Clave E

34.



$M = \text{sen}\theta\cos\theta$

$M = \dfrac{y}{r}\cdot\dfrac{x}{r} = \dfrac{xy}{r^2} = \dfrac{xy}{x^2+y^2}$

$M = \dfrac{xy}{x^2+y^2} = \dfrac{\left(-\frac{R}{2}\right)\left(-\left(\frac{\sqrt{3}+2}{2}\right)R\right)}{\left(\frac{-R}{2}\right)^2 + \left(-\left(\frac{\sqrt{3}+2}{2}\right)R\right)^2}$

$M = \dfrac{R^2\left(\frac{(\sqrt{3}+2)}{4}\right)}{R^2\left(\frac{1}{4} + \frac{7+4\sqrt{3}}{4}\right)}$

$M = \dfrac{(\sqrt{3}+2)/4}{(8+4\sqrt{3})/4} = \dfrac{2+\sqrt{3}}{8+4\sqrt{3}}$

$M = 1/4$

Clave D

REDUCCIÓN AL PRIMER CUADRANTE

APLICAMOS LO APRENDIDO
(página 58) Unidad 3

1. $P=\dfrac{\cos 330^\circ \cot 300^\circ \csc 135^\circ}{\sec 315^\circ \operatorname{sen}300^\circ \tan 330^\circ}$

$P=\dfrac{\cos(360^\circ-30^\circ)\cot(360^\circ-60^\circ)\csc(90^\circ+45^\circ)}{\sec(360^\circ-45^\circ)\operatorname{sen}(360^\circ-60^\circ)\tan(360^\circ-30^\circ)}$

$P=\dfrac{(\cos 30^\circ)(-\cot 60^\circ)(\sec 45^\circ)}{(\sec 45^\circ)(-\operatorname{sen}60^\circ)(-\tan 30^\circ)}$

$\therefore P=-\dfrac{\left(\frac{\sqrt3}{2}\right)\left(\frac{\sqrt3}{3}\right)(\sqrt2)}{(\sqrt2)\left(\frac{\sqrt3}{2}\right)\left(\frac{\sqrt3}{3}\right)}=-1$

Clave C

2. α y θ son complementarios $\Rightarrow \alpha+\theta=90^\circ$

$M=\dfrac{\operatorname{sen}(\alpha+2\theta)\tan(2\alpha+3\theta)}{\cos(2\alpha+\theta)\tan(4\alpha+3\theta)}$

$M=\dfrac{\operatorname{sen}((\alpha+\theta)+\theta)\tan(2(\alpha+\theta)+\theta)}{\cos((\alpha+\theta)+\alpha)\tan(3(\alpha+\theta)+\alpha)}$

$M=\dfrac{\operatorname{sen}(90^\circ+\theta)\tan(180^\circ+\theta)}{\cos(90^\circ+\alpha)\tan(270^\circ+\alpha)}$

$M=\dfrac{\cos\theta\tan\theta}{(-\operatorname{sen}\alpha)(-\cot\alpha)}$

$M=\dfrac{\cos\theta\tan\theta}{\operatorname{sen}\alpha\cot\alpha}=\dfrac{\cos\theta.\frac{\operatorname{sen}\theta}{\cos\theta}}{\operatorname{sen}\alpha.\frac{\cos\alpha}{\operatorname{sen}\alpha}}$

$M=\dfrac{\operatorname{sen}\theta}{\cos\alpha}=\dfrac{\operatorname{sen}(90^\circ-\alpha)}{\cos\alpha}$

$M=\dfrac{\cos\alpha}{\cos\alpha}=1\Rightarrow M=1$

Clave A

3. $K=\dfrac{\operatorname{sen}390^\circ-\tan 2280^\circ}{\cos 1560^\circ}$

$K=\dfrac{\operatorname{sen}(360^\circ.1+30^\circ)-\tan(360^\circ.6+120^\circ)}{\cos(360^\circ.4+120^\circ)}$

$K=\dfrac{\operatorname{sen}30^\circ-\tan 120^\circ}{\cos 120^\circ}$

$K=\dfrac{\operatorname{sen}30^\circ-(-\tan 60^\circ)}{-\cos 60^\circ}$

$K=-\dfrac{\operatorname{sen}30^\circ+\tan 60^\circ}{\cos 60^\circ}=-\dfrac{\left(\frac12\right)+(\sqrt3)}{\left(\frac12\right)}$

$K=-(1+2\sqrt3)$

$\therefore K=-1-2\sqrt3$

Clave D

4. $P=\operatorname{sen}1920^\circ\,[\operatorname{sen}(-60^\circ)-\cos(-45^\circ)]$
$P=[\operatorname{sen}(360^\circ.5+120^\circ)]\,[-\operatorname{sen}60^\circ-\cos 45^\circ]$
$P=\operatorname{sen}120^\circ\,(-\operatorname{sen}60^\circ-\cos 45^\circ)$

$P=\dfrac{\sqrt3}{2}\left(-\dfrac{\sqrt3}{2}-\dfrac{\sqrt2}{2}\right)=\dfrac{\sqrt3}{2}\left(\dfrac{-\sqrt3-\sqrt2}{2}\right)$

$\therefore P=\dfrac{-3-\sqrt6}{4}$

Clave E

5. $R=\dfrac{\operatorname{sen}140^\circ+\cos 50^\circ}{\cos 130^\circ}$

$R=\dfrac{\operatorname{sen}(180^\circ-40^\circ)+\cos 50^\circ}{\cos(180^\circ-50^\circ)}$

$R=\dfrac{\operatorname{sen}40^\circ+\cos 50^\circ}{-\cos 50^\circ}=\dfrac{(\cos 50^\circ)+\cos 50^\circ}{-\cos 50^\circ}$

$R=-\dfrac{2\cos 50^\circ}{\cos 50^\circ}=-2$

$\therefore R=-2$

Clave A

6. $E=\dfrac{7\operatorname{sen}40^\circ+3\cos 50^\circ}{\operatorname{sen}140^\circ}$

$E=\dfrac{7\operatorname{sen}40^\circ+3\cos(90^\circ-40^\circ)}{\operatorname{sen}(180^\circ-40^\circ)}$

$E=\dfrac{7\operatorname{sen}40^\circ+3\operatorname{sen}40^\circ}{\operatorname{sen}40^\circ}=\dfrac{10\operatorname{sen}40^\circ}{\operatorname{sen}40^\circ}$

$\therefore E=10$

Clave E

7. $S=\sqrt{15+10\sqrt{2\operatorname{sen}150^\circ}}$

$S=\sqrt{15+10\sqrt{2\operatorname{sen}30^\circ}}$

$S=\sqrt{15+10\sqrt{2\left(\frac12\right)}}$

$S=\sqrt{15+10\sqrt1}=\sqrt{15+10}$

$S=\sqrt{25}=5$

$\therefore S=5$

Clave A

8. $A=\sqrt[3]{24+\sqrt3\,(\tan 600^\circ)}$

$\Rightarrow \tan 600^\circ=\tan(360^\circ+240^\circ)=\tan 240^\circ$
$=\tan(180^\circ+60^\circ)=\tan 60^\circ$

$\tan 600^\circ=\tan 60^\circ=\sqrt3$

$A=\sqrt[3]{24+\sqrt3(\sqrt3)}$

$A=\sqrt[3]{24+3}=\sqrt[3]{27}$

$\therefore A=3$

Clave D

9. $\dfrac{\operatorname{sen}(\pi-\alpha)\cos\left(\frac{\pi}{2}+\alpha\right)\tan(\pi-\alpha)}{\cot\left(\frac{\pi}{2}-\alpha\right)\sec\left(\frac{\pi}{2}+\alpha\right)\csc(\pi-\alpha)}$

$=\dfrac{(+\operatorname{sen}\alpha)(-\operatorname{sen}\alpha)(-\tan\alpha)}{(+\tan\alpha)(-\csc\alpha)(+\csc\alpha)}$

$=\dfrac{+\operatorname{sen}^2\alpha}{-\csc^2\alpha}=-\operatorname{sen}^2\alpha\,.\,\operatorname{sen}^2\alpha=-\operatorname{sen}^4\alpha$

Clave E

10. $A=\dfrac{\cos\left(\frac{3\pi}{2}+x\right)}{\operatorname{sen}(-x)}-\dfrac{\tan(2\pi+x)}{\tan(-x)}$

$A=\dfrac{\operatorname{sen}x}{-\operatorname{sen}x}-\dfrac{\tan x}{-\tan x}$

$A=-\dfrac{\operatorname{sen}x}{\operatorname{sen}x}+\dfrac{\tan x}{\tan x}=-1+1=0$

$\therefore A=0$

Clave E

11. $P=\dfrac{-\operatorname{sen}x}{-\operatorname{sen}x}+\dfrac{\cos x}{\cos(\pi-x)}+\tan 0^\circ$

$P=1+\dfrac{\cos x}{-\cos x}+0$

$P=1-1+0$
$P=0$

Clave D

12. $Q = \dfrac{\text{sen}(270° - (30° + x)) + \cos(180° + (30° + x))}{\cos(30 + x)}$

$Q = \dfrac{-\cos(30° + x) - \cos(30° + x)}{\cos(30° + x)}$

$Q = \dfrac{-2\cos(30° + x)}{\cos(30° + x)}$

$Q = -2$

Clave C

13. $E = \dfrac{\text{sen}(180° - 10°)\cos(180° + 10°)\cos(360° - 10°)}{\cos(270° + 10°)\csc(90° + 10°)\csc(270° - 10°)}$

$E = \dfrac{\text{sen}10°(-\cos 10°)\cos 10°}{\text{sen}10° \sec 10°(-\sec 10°)}$

$E = \dfrac{-\cos^2 10°}{-\sec^2 10°}$

$E = \dfrac{\cos^2 10°}{\sec^2 10°}$

$E = \cos^4 10°$

$E = a^4$

Clave A

14. $M = \dfrac{\text{sen}\left(26\pi - \frac{\pi}{3}\right)\tan\left(9\pi + \frac{\pi}{6}\right)\text{sen}\left(8\pi + \frac{\pi}{4}\right)}{\cos\left(3\pi - \frac{\pi}{4}\right)\csc\left(15\pi - \frac{\pi}{3}\right)\cot\left(13\pi - \frac{\pi}{6}\right)}$

$M = \dfrac{\left(-\text{sen}\frac{\pi}{3}\right)\left(\tan\frac{\pi}{6}\right)\left(\sec\frac{\pi}{4}\right)}{\left(-\cos\frac{\pi}{4}\right)\left(\csc\frac{\pi}{3}\right)\left(-\cot\frac{\pi}{6}\right)}$

$M = -\dfrac{\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{3} \cdot \sqrt{2}}{\frac{\sqrt{2}}{2} \cdot \frac{2\sqrt{3}}{3} \cdot \sqrt{3}} \Rightarrow M = -\dfrac{1}{2}$

Clave B

PRACTIQUEMOS

Nivel 1 (página 60) Unidad 3

Comunicación matemática

1. a. Coterminales b. Agudo c. Secante
   d. Suplementario e. Radial f. Cuadrantales
   g. Beta h. Coseno

   ∴ La palabra sombreada es Euclides.

2.

Razonamiento y demostración

3. $C = \text{sen}150°\cos240°$

$C = \text{sen}(180° - 30°)\cos(180° + 60°)$

$C = \text{sen}(30°) \cdot -\cos(60°)$

$\therefore C = \dfrac{1}{2} \cdot \left(-\dfrac{1}{2}\right) = -\dfrac{1}{4}$

Clave D

4. $L = \tan2310°\text{sen}1935°$

$L = \tan(360°k + 150°)\text{sen}(360°k + 135°)$

$L = \tan150°\text{sen}135°$

$L = \tan(180° - 30°)\ \text{sen}(180° - 45°)$

$L = -\tan30°\text{sen}45°$

$\therefore L = -\dfrac{\sqrt{3}}{3} \cdot \dfrac{\sqrt{2}}{2} = -\dfrac{\sqrt{6}}{6}$

Clave D

5. $C = \dfrac{\text{sen}2640°}{\cos 3120°} = \dfrac{\text{sen}(360°k + 120°)}{\cos(360°k + 240°)}$

$C = \dfrac{\text{sen}120°}{\cos 240°}$

$C = \dfrac{\text{sen}(180° - 60°)}{\cos(180° + 60°)} = \dfrac{\text{sen}60°}{-\cos 60°} = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

Clave B

6. $C = \text{sen}135°\cos217°\tan307°$

$C = \text{sen}(180° - 45°)\cos(180° + 37°)\tan(360° - 53°)$

$C = \text{sen}45°(-\cos37°)\ (-\tan53°)$

$C = \text{sen}45° \cdot \dfrac{4}{5} \cdot \dfrac{4}{3}$

$\therefore C = \dfrac{\sqrt{2}}{2} \cdot \dfrac{16}{15} = \dfrac{8\sqrt{2}}{15}$

Clave A

7. $L = \dfrac{\tan 150°\text{sen}120°}{\cos 225°}$

$L = \dfrac{\tan(180° - 30°)\text{sen}(180° - 60°)}{\cos(180° + 45°)}$

$L = \dfrac{-\tan 30°\text{sen}60°}{-\cos 45°}$

$L = \dfrac{\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}}$

$\therefore L = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

Clave E

8. $L = \tan(-120°)\cos(-300°)$

$L = -\tan120°\cos(300°)$

$L = -\tan(180° - 60°)\cos(360° - 60°)$

$L = -(-\tan60°)\cos60°$

$L = \tan60°\cos60° = \sqrt{3} \cdot \dfrac{1}{2} = \dfrac{\sqrt{3}}{2}$

Clave C

9. $C = \text{sen}(-45°)\tan(-60°)\cos(-30°)$

$C = -\text{sen}45° - \tan60°\cos30°$

$C = \text{sen}45°\tan60°\cos30°$

$\therefore C = \dfrac{\sqrt{2}}{2} \cdot \sqrt{3} \cdot \dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{2}}{4}$

Clave C

10. $L = \dfrac{\text{sen}112°}{\text{sen}68°} + \dfrac{\cos 132°}{\cos 48°} + \dfrac{\tan 310°}{\tan 50°}$

$L = \dfrac{\text{sen}68°}{\text{sen}68°} - \dfrac{\cos 48°}{\cos 48°} - \dfrac{\tan 50°}{\tan 50°}$

$\therefore L = 1 - 1 - 1 = -1$

Clave D

Nivel 2 (página 61) Unidad 3

Comunicación matemática

11. I. V II. V III. V

12.

Razonamiento y demostración

13. $L = \dfrac{\text{sen}140°\cos200°\tan160°}{\text{sen}320°\cos340°\tan200°} = \dfrac{\text{sen}40°(-\cos20°)(-\tan20°)}{-\text{sen}40°\cos20°\tan20°}$

$\therefore\ L = \dfrac{1}{-1} = -1$

Clave B

14. $L = \text{sen}121\frac{\pi}{4}\cos97\frac{\pi}{3}\sec77\frac{\pi}{6}$

$L = \text{sen}\left(360°k + \frac{\pi}{4}\right)\cos\left(360°k + \frac{\pi}{3}\right)\sec\left(360°k + \frac{5\pi}{6}\right)$

$L = \text{sen}\frac{\pi}{4}\ .\ \cos\frac{\pi}{3}\ .\ \sec\frac{5\pi}{6}$

$L = \frac{\sqrt{2}}{2}\ .\ \frac{1}{2}\ .\ \sec\left(\pi - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{4}\left(-\sec\frac{\pi}{6}\right)$

$\therefore\ L = \frac{\sqrt{2}}{4}\ .\ \left(-\frac{2}{\sqrt{3}}\right) = -\frac{\sqrt{2}}{2\sqrt{3}}\ .\ \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{6}}{6}$

Clave D

15. $J = \text{sen}(x - 270°)\sec(x - 180°)$

$J = -\text{sen}(270° - x)\sec(180° - x) = -(-\cos x)(-\sec x)$

$\therefore\ J = \cos x(-\sec x) = -1$

Clave B

16. $C = \dfrac{\tan(x - 270°)}{\cot(x - 180°)} = \dfrac{-\tan(270° - x)}{-\cot(180° - x)}$

$C = \dfrac{\tan(270° - x)}{\cot(180° - x)} = \dfrac{\cot x}{-\cot x} = -1$

Clave B

17. $C = \text{sen}(270° + x)\sec(180° + x)\tan(90° + x)$

$C = (-\cos x)(-\sec x)(-\cot x) = 1(-\cot x) = -\cot x$

Clave D

18. $J = \dfrac{\text{sen}(180° + x)\tan(270° - x)}{\cot(360° - x)}$

$\therefore\ J = \dfrac{(-\text{sen}x)\cot x}{(-\cot x)} = \text{sen}x$

Clave A

19. $C = \dfrac{\text{sen}(90° + x)}{\cos(180° - x)} + \dfrac{\text{sen}(360° - x)}{\cos(270° - x)}$

$C = \dfrac{\cos x}{-\cos x} + \dfrac{-\text{sen}x}{-\text{sen}x}$

$\therefore\ C = -1 + 1 = 0$

Clave B

20. $J = \dfrac{\text{sen}(180° - x)}{\text{sen}(-x)} + \dfrac{\cos(180° + x)}{\cos(-x)}$

$J = \dfrac{\text{sen}x}{-\text{sen}x} + \dfrac{-\cos x}{\cos x}$

$J = -1 - 1 = -2$

Clave E

Nivel 3 (página 61) Unidad 3

Comunicación matemática

21. I. V II. V III. V

22. $\tan\theta = 1$ $\quad \text{sen}\theta = \dfrac{\sqrt{2}}{2}$ $\quad \cot\theta = 1$

Razonamiento y demostración

23. $J = \dfrac{\text{sen}(x + \pi)\cos\left(\frac{\pi}{2} + x\right)}{\sec\left(\frac{3\pi}{2} + x\right)}$

$J = \dfrac{(-\text{sen}x)(-\text{sen}x)}{\csc x}$

$J = \dfrac{\text{sen}x\,\text{sen}x}{\frac{1}{\text{sen}x}}$ $\quad\therefore\ J = \text{sen}^3x$

Clave C

24. $C = \tan(\pi - x)\tan\left(\frac{3\pi}{2} - x\right)\text{sen}\left(\frac{\pi}{2} + x\right)$

$C = -\tan x\cot x\cos x$

$\therefore\ C = -\cos x$

Clave D

25. $J = \dfrac{\text{sen}\left(231\frac{\pi}{2} + x\right)\tan(125\pi + x)}{\cos(132\pi - x)}$

$J = \dfrac{\text{sen}\left(360°k + \frac{3\pi}{2} + x\right)\tan(360°k + \pi + x)}{\cos(360°k - x)}$

$J = \dfrac{\text{sen}\left(\frac{3\pi}{2} + x\right)\tan(\pi + x)}{\cos(-x)}$

$J = \dfrac{-\cos x\tan x}{\cos x}$ $\quad\therefore\ J = -\tan x$

Clave B

26. $C = \dfrac{\tan\left(2001\frac{\pi}{2} - x\right)\sec(2002\pi - x)}{\tan\left(2003\frac{\pi}{2} - x\right)}$

$C = \dfrac{\tan\left(360°k + \frac{\pi}{2} - x\right)\sec(360°k + 2\pi - x)}{\tan\left(360°k - \frac{\pi}{2} - x\right)}$

$C = \dfrac{\tan\left(\frac{\pi}{2} - x\right)\sec(2\pi - x)}{\tan\left(-\frac{\pi}{2} - x\right)} = \dfrac{\cot x\sec x}{\tan\left(\frac{\pi}{2} + x\right)} = \dfrac{\cot x\sec x}{-\cot x}$

$\therefore\ C = -\sec x$

Clave B

27. $J = \dfrac{\text{sen}(A + B)}{\text{sen}C} + \dfrac{\tan(B + C)}{\tan A} + \dfrac{\cos(A + C)}{\cos B}$

$J = \dfrac{\text{sen}(180° - C)}{\text{sen}C} + \dfrac{\tan(180° - A)}{\tan A} + \dfrac{\cos(180° - B)}{\cos B}$

$J = \dfrac{\text{sen}C}{\text{sen}C} - \dfrac{\tan A}{\tan A} - \dfrac{\cos B}{\cos B}$ $\quad\therefore\ J = 1 - 1 - 1 = -1$

Clave C

28. $C = \dfrac{\text{sen}(\alpha - \beta)}{\text{sen}(\beta - \alpha)} + \dfrac{\cos(\beta - \theta)}{\cos(\theta - \beta)} + \dfrac{\tan(\theta - \alpha)}{\tan(\alpha - \theta)}$

$C = \dfrac{-\text{sen}(\beta - \alpha)}{\text{sen}(\beta - \alpha)} + \dfrac{\cos(\theta - \beta)}{\cos(\theta - \beta)} - \dfrac{\tan(\alpha - \theta)}{\tan(\alpha - \theta)}$

$\therefore\ C = -1 + 1 - 1 = -1$

Clave B

29. $\text{sen}20° = n$

$C = \text{sen}200°\tan340°\cos160°$

$C = \text{sen}(180° + 20°)\tan(360° - 20°)\cos(180° - 20°)$

$C = -\text{sen}20°(-\tan20°)(-\cos20°) = (-\text{sen}20°)\tan20°\cos20°$

$C = (-\text{sen}20°)\dfrac{\text{sen}20°}{\cos20°}\cos20°$ $\quad\therefore\ C = -\text{sen}^2 20° = -n^2$

Clave B

30. $\tan10° = n$

$L = \tan190°\text{sen}170°\cos350°$

$L = \tan(180° + 10°)\text{sen}(180° - 10°)\cos(360° - 10°)$

$L = \tan10°\text{sen}10°\cos10°$

$L = \dfrac{\text{sen}10°}{\cos10°}\text{sen}10°\cos10°\dfrac{\cos10°}{\cos10°} = (\tan10°)^2\dfrac{1}{\sec^2 10°} = n^2\dfrac{1}{1 + \tan^2 10°}$

$\therefore\ L = \dfrac{n^2}{n^2 + 1}$

Clave D

IDENTIDADES TRIGONOMÉTRICAS

APLICAMOS LO APRENDIDO (página 63) Unidad 3

1. Para demostrar, escogemos el miembro más operativo:

$$(\sec x + \tan x - 1)(1 + \sec x - \tan x) = 2\tan x$$
$$(\sec x - (1 - \tan x))(\sec x + (1 - \tan x)) = 2\tan x$$
$$(\sec x)^2 - (1 - \tan x)^2 = 2\tan x$$
$$1 + \tan^2 x - (1 + \tan^2 x - 2\tan x) = 2\tan x$$
$$1 + \tan^2 x - 1 - \tan^2 x + 2\tan x = 2\tan x$$
$$2\tan x = 2\tan x$$

2. Elevamos al cuadrado las dos igualdades:

$(\operatorname{sen}\theta + \csc\theta)^2 = (m)^2$

$\operatorname{sen}^2\theta + \csc^2\theta + \underbrace{2\operatorname{sen}\theta\csc\theta}_{(1)} = m^2$

$\operatorname{sen}^2\theta + \csc^2\theta = m^2 - 2$ ... (I)

$(\operatorname{sen}\theta - \csc\theta)^2 = n^2$

$\operatorname{sen}^2\theta + \csc^2\theta - 2 = n^2$

$\operatorname{sen}^2\theta + \csc^2\theta = n^2 + 2$ ... (II)

Igualamos (I) y (II):

$m^2 - 2 = n^2 + 2$

$m^2 - n^2 = 4$

Clave C

3. Desarrollamos la expresión:

$$\frac{1}{1+\cos\beta} + \frac{1}{1-\cos\beta} = \frac{25}{8}$$
$$\frac{(1-\cos\beta)+(1+\cos\beta)}{(1+\cos\beta)(1-\cos\beta)} = \frac{25}{8}$$
$$\frac{2}{1-\cos^2\beta} = \frac{25}{8} \Rightarrow \frac{16}{25} = 1 - \cos^2\beta$$
$$\frac{16}{25} = \operatorname{sen}^2\beta$$
$$\operatorname{sen}\beta = \sqrt{\frac{16}{25}} = \frac{4}{5}$$
$$\therefore\ \csc\beta = \frac{1}{\operatorname{sen}\beta} = \frac{5}{4}$$

Clave A

4. Desarrollamos la expresión:

$$M = \frac{\operatorname{sen}^3 x - \cos^3 x}{\cos x - \operatorname{sen} x} + \operatorname{sen}x\cos x$$
$$M = \frac{(\operatorname{sen}x - \cos x)(\operatorname{sen}^2 x + \operatorname{sen}x\cos x + \cos^2 x)}{-(\operatorname{sen}x - \cos x)} + \operatorname{sen}x\cos x$$
$$M = -(\operatorname{sen}^2 x + \cos^2 x + \operatorname{sen}x\cos x) + \operatorname{sen}x\cos x$$
$$M = -1 - \operatorname{sen}x\cos x + \operatorname{sen}x\cos x$$
$$\therefore\ M = -1$$

Clave D

5. $P = \sqrt[3]{\sec^2\theta + \csc^2\theta - 1}$

$$P = \sqrt[3]{\frac{1}{\cos^2\theta} + \frac{1}{\operatorname{sen}^2\theta} - 1}$$
$$P = \sqrt[3]{\frac{\operatorname{sen}^2\theta + \cos^2\theta}{\operatorname{sen}^2\theta\cos^2\theta} - 1}$$
$$= \sqrt[3]{\frac{1}{(\operatorname{sen}\theta\cos\theta)^2} - 1}$$
$$P = \sqrt[3]{\frac{1}{1/9} - 1} = \sqrt[3]{9-1} = \sqrt[3]{8} = 2 \qquad \therefore\ P = 2$$

Clave E

6. $\cos\theta = k - \operatorname{sen}\theta$

$(\cos\theta + \operatorname{sen}\theta) = k$

$(\operatorname{sen}\theta + \cos\theta)^2 = (k^2)$

$\operatorname{sen}^2\theta + 2\operatorname{sen}\theta\cos\theta + \cos^2\theta = k^2$

$1 + 2\operatorname{sen}\theta\cos\theta = k^2$

$\therefore\ \operatorname{sen}\theta\cos\theta = \dfrac{k^2-1}{2}$

Clave D

7. Desarrollamos la expresión:

$$R = \frac{\cos^4\alpha - \operatorname{sen}^4\alpha}{\operatorname{sen}\alpha - \cos\alpha} + \operatorname{sen}\alpha$$
$$R = \frac{(\cos^2\alpha - \operatorname{sen}^2\alpha)(\cos^2\alpha + \operatorname{sen}^2\alpha)}{\operatorname{sen}\alpha - \cos\alpha} + \operatorname{sen}\alpha$$
$$R = \frac{(\cos\alpha - \operatorname{sen}\alpha)(\cos\alpha + \operatorname{sen}\alpha)}{-(\cos\alpha - \operatorname{sen}\alpha)} + \operatorname{sen}\alpha$$
$$R = -\cos\alpha - \operatorname{sen}\alpha + \operatorname{sen}\alpha = -\cos\alpha$$
$$\therefore\ R = -\cos\alpha$$

Clave C

8. $P = \dfrac{1}{\sec x + \tan x} + \dfrac{1}{\sec x - \tan x}$

$$P = \frac{(\sec x - \tan x) + (\sec x + \tan x)}{(\sec x + \tan x)(\sec x - \tan x)}$$
$$P = \frac{2\sec x}{(\sec^2 x - \tan^2 x)} = \frac{2\sec x}{1}$$
$$\therefore\ P = 2\sec x$$

Clave E

9. $m\sec x = \cos x$ ...(I)

$n\csc x = \operatorname{sen}x$ ...(II)

De (I): $m\sec x = \cos x$

$\dfrac{m}{\cos x} = \cos x \Rightarrow m = \cos^2 x$

De (II): $n\csc x = \operatorname{sen}x$

$\dfrac{n}{\operatorname{sen}x} = \operatorname{sen}x \Rightarrow n = \operatorname{sen}^2 x$

Piden: $m + n$

$m + n = (\cos^2 x) + (\operatorname{sen}^2 x) = 1$

$\therefore\ m + n = 1$

Clave B

10. $P = \left(\tan x + \dfrac{\cos x}{1+\operatorname{sen}x}\right)\left(\cot x + \dfrac{\operatorname{sen}x}{1+\cos x}\right)$

$$P = \left(\frac{\operatorname{sen}x}{\cos x} + \frac{\cos x}{1+\operatorname{sen}x}\right)\left(\frac{\cos x}{\operatorname{sen}x} + \frac{\operatorname{sen}x}{1+\cos x}\right)$$
$$P = \left(\frac{\operatorname{sen}x + \operatorname{sen}^2 x + \cos^2 x}{\cos x(1+\operatorname{sen}x)}\right)\left(\frac{\cos x + \cos^2 x + \operatorname{sen}^2 x}{\operatorname{sen}x(1+\cos x)}\right)$$
$$P = \left(\frac{1+\operatorname{sen}x}{\cos x(1+\operatorname{sen}x)}\right)\left(\frac{1+\cos x}{\operatorname{sen}x(1+\cos x)}\right)$$
$$P = \left(\frac{1}{\cos x}\right)\left(\frac{1}{\operatorname{sen}x}\right) = (\sec x)(\csc x)$$
$$\therefore\ P = \sec x\csc x$$

Clave E

11. $x = 2\tan\theta \Rightarrow x^2 = 4\tan^2\theta$

Piden:

$$\sqrt{4+x^2} = \sqrt{4+4\tan^2\theta} = \sqrt{4(1+\tan^2\theta)}$$
$$\sqrt{4+x^2} = \sqrt{4(\sec^2\theta)} = 2|\sec\theta|$$
$$\therefore\ \sqrt{4+x^2} = 2|\sec\theta|$$

Clave E

12. $\tan\alpha + \cot\alpha = a$ ...(I)

$\tan\alpha - \cot\alpha = b$ ...(II)

Piden: $a^2 - b^2$

$a^2 - b^2 = (a + b)(a - b)$

De (I) y (II):

$a + b = 2\tan\alpha$

$a - b = 2\cot\alpha$

Reemplazando tenemos:

$a^2 - b^2 = (2\tan\alpha)(2\cot\alpha)$

$a^2 - b^2 = 4\underbrace{\tan\alpha\cot\alpha}_{1}$

$\therefore a^2 - b^2 = 4$

Clave B

13. Invertimos la igualdad:

$$\frac{m}{\text{sen}\alpha} = \frac{n}{\cos\alpha} = \frac{p}{\text{sen}\alpha\cos\alpha} = k$$

$$\frac{\text{sen}\alpha}{m} = \frac{\cos\alpha}{n} = \frac{\text{sen}\alpha\cos\alpha}{p} = \frac{1}{k}$$

$$\Rightarrow \frac{1}{m} = \frac{1}{k\,\text{sen}\alpha};\ \frac{1}{n} = \frac{1}{k\cos\alpha};\ \frac{1}{p} = \frac{1}{k\,\text{sen}\alpha\cos\alpha}$$

$$\frac{1}{m^2} + \frac{1}{n^2} = \frac{1}{k^2\text{sen}^2\alpha} + \frac{1}{k^2\cos^2\alpha}$$

$$\frac{1}{m^2} + \frac{1}{n^2} = \frac{k^2(\cos^2\alpha + \text{sen}^2\alpha)}{k^2\text{sen}^2\alpha\cos^2\alpha k^2}$$

$$\frac{1}{m^2} + \frac{1}{n^2} = \frac{1}{(k\,\text{sen}\alpha\cos\alpha)^2}$$

$$\Rightarrow \frac{1}{m^2} + \frac{1}{n^2} = \frac{1}{p^2}$$

Clave B

14. $\cot^2x - \cos^2x = \cot^2x\,.\,R$

$$\frac{\cos^2x}{\text{sen}^2x} - \cos^2x = \frac{\cos^2x}{\text{sen}^2x}R$$

$\cos^2x - \text{sen}^2x\cos^2x = \cos^2x\,R$

$1 - \text{sen}^2x = R$

$\therefore \cos^2x = R$

Clave B

PRACTIQUEMOS

Nivel 1 (página 65) Unidad 3

Comunicación matemática

1.

2. I. $\text{sen}\alpha = \dfrac{1}{\csc\alpha} \Rightarrow \text{sen}\alpha\csc\alpha = 1$ (V)

II. $\cos^2\alpha = (1 + \text{sen}\alpha)(1 - \text{sen}\alpha)$

$\cos^2\alpha = 1 - \text{sen}^2\alpha$ (V)

III. $\tan\alpha = \dfrac{\csc\alpha}{\sec\alpha}$

$\tan\alpha = \dfrac{1/\text{sen}\alpha}{1/\cos\alpha} = \dfrac{\cos\alpha}{\text{sen}\alpha}$ (F)

IV. $\text{sen}^2\alpha = (1 + \cos\alpha)(1 + \cos\alpha)$

$\text{sen}^2\alpha = (1 + \cos\alpha)^2$

$\text{sen}^2\alpha = 1 + \cos^2\alpha + 2\cos\alpha$ (F)

V. $\cot\alpha = \dfrac{\csc\alpha}{\sec\alpha} = \dfrac{\cos\alpha}{\text{sen}\alpha}$ (V)

Clave D

Razonamiento y demostración

3. $$T = \frac{\text{sen}^2\alpha - \cos^2\alpha}{\text{sen}\alpha + \cos\alpha} + \cos\alpha$$

$$T = \frac{(\text{sen}\alpha + \cos\alpha)(\text{sen}\alpha - \cos\alpha)}{\text{sen}\alpha + \cos\alpha} + \cos\alpha$$

$T = \text{sen}\alpha - \cos\alpha + \cos\alpha$

$T = \text{sen}\alpha$

Clave A

4. $$S = \frac{\sec\theta - \cos\theta}{\tan\theta} = \frac{\dfrac{1}{\cos\theta} - \cos\theta}{\tan\theta}$$

$$\Rightarrow S = \frac{\dfrac{1 - \cos^2\theta}{\cos\theta}}{\dfrac{\text{sen}\theta}{\cos\theta}} = \frac{\text{sen}^2\theta}{\text{sen}\theta}$$

$\therefore S = \text{sen}\theta$

Clave A

5. $$V = \frac{1 + \cos\theta}{\text{sen}\theta} + \frac{\text{sen}\theta}{1 + \cos\theta}$$

$$V = \frac{(1 + \cos\theta)^2 + \text{sen}^2\theta}{\text{sen}\theta(1 + \cos\theta)}$$

$$V = \frac{1 + 2\cos\theta + (\cos^2\theta + \text{sen}^2\theta)}{\text{sen}\theta(1 + \cos\theta)}$$

$$V = \frac{2 + 2\cos\theta}{\text{sen}\theta(1 + \cos\theta)} = \frac{2(1 + \cos\theta)}{\text{sen}\theta(1 + \cos\theta)}$$

$V = 2\,.\,\dfrac{1}{\text{sen}\theta}$ $\quad\therefore V = 2\csc\theta$

Clave E

6. $C = (1 - \text{sen}^2\theta)\tan\theta - \text{sen}\theta\cos\theta$

$C = \cos^2\theta\tan\theta - \text{sen}\theta\cos\theta$

$C = \cos^2\theta\,.\,\dfrac{\text{sen}\theta}{\cos\theta} - \text{sen}\theta\cos\theta$

$C = \text{sen}\theta\cos\theta - \text{sen}\theta\cos\theta = 0$

Clave B

7. $$L = \frac{2\csc\theta + \cos\theta}{2\sec\theta + \text{sen}\theta} = \frac{2\,.\,\dfrac{1}{\text{sen}\theta} + \cos\theta}{2\,.\,\dfrac{1}{\cos\theta} + \text{sen}\theta}$$

$$L = \frac{\dfrac{2 + \text{sen}\theta\cos\theta}{\text{sen}\theta}}{\dfrac{2 + \text{sen}\theta\cos\theta}{\cos\theta}}$$

$$L = \frac{(2 + \text{sen}\theta\cos\theta)\cos\theta}{(2 + \text{sen}\theta\cos\theta)\text{sen}\theta}$$

$$L = \frac{\cos\theta}{\text{sen}\theta} = \cot\theta$$

Clave C

8. $$I = \frac{\cos^2x}{\csc^2x - 1} + \frac{\text{sen}^2x}{\sec^2x - 1}$$

$\csc^2x - \cot^2x = 1 \Rightarrow \csc^2x - 1 = \cot^2x$

$\sec^2x - \tan^2x = 1 \Rightarrow \sec^2x - 1 = \tan^2x$

$$I = \frac{\cos^2x}{\cot^2x} + \frac{\text{sen}^2x}{\tan^2x}$$

$$I = \frac{\cos^2x\tan^2x + \text{sen}^2x\cot^2x}{\cot^2x\tan^2x}$$

$$I = \cos^2x\frac{\text{sen}^2x}{\cos^2x} + \text{sen}^2x\frac{\cos^2x}{\text{sen}^2x}$$

$I = \text{sen}^2x + \cos^2x = 1$

Clave A

9. $E = (\tan x + \cot x)\cos x$

$$E = \left(\frac{\text{sen}x}{\cos x} + \frac{\cos x}{\text{sen}x}\right)\cos x$$

$$E = \left(\frac{\text{sen}^2x + \cos^2x}{\text{sen}x\cos x}\right)\cos x$$

$$\therefore E = \frac{1}{\text{sen}x} = \csc x$$

Clave E

10. $S = (\sec x + \tan x)(1 - \text{sen}x)$

$$S = \left(\frac{1}{\cos x} + \frac{\text{sen}x}{\cos x}\right)(1 - \text{sen}x)$$

$$S = \left(\frac{1 + \text{sen}x}{\cos x}\right)(1 - \text{sen}x)$$

$$S = \frac{1 - \text{sen}^2x}{\cos x} = \frac{\cos^2x}{\cos x}$$

$\therefore S = \cos x$

Clave A

Resolución de problemas

11. $a = \sqrt{\tan\theta\sqrt{\tan\theta\sqrt{\tan\theta\sqrt{...}}}}$

$a = \sqrt{\tan\theta\,.\,a}$

$a^2 = \tan\theta a \Rightarrow \tan\theta = a$

Despejamos k:

$$k = \frac{\sec\theta + 3\tan\theta + 2}{\csc\theta + 2\cot\theta + 3}$$

$$k = \frac{\dfrac{1}{\cos\theta} + 3\dfrac{\text{sen}\theta}{\cos\theta} + 2}{\dfrac{1}{\text{sen}\theta} + 2\dfrac{\cos\theta}{\text{sen}\theta} + 3}$$

$$k = \frac{\dfrac{1 + 3\text{sen}\theta + 2\cos\theta}{\cos\theta}}{\dfrac{1 + 2\cos\theta + 3\text{sen}\theta}{\text{sen}\theta}}$$

$$k = \frac{\text{sen}\theta}{\cos\theta} = \tan\theta = a$$

$\therefore k = a$

Clave E

12. Desarrollamos la expresión:

$(3\text{sen}x + \cos x)^2 + (\text{sen}x + 3\cos x)^2 = a - b\text{sen}x\cos x$

$9\text{sen}^2x + 6\text{sen}x\cos x + \cos^2x + \text{sen}^2x +$

$6\text{sen}x\cos x + 9\cos^2x = a - b\text{sen}x\cos x$

$10\text{sen}^2x + 12\text{sen}x\cos x + 10\cos^2x$

$= a - b\text{sen}x\cos x$

$10(\text{sen}^2x + \cos^2x) + 12\text{sen}x\cos x$

$= a - b\text{sen}x\cos x$

$10 + 12\text{sen}x\cos x = a - b\text{sen}x\cos x$

$\therefore a = 10 \wedge b = -12$

Hallamos el valor de:

$$M = \frac{a + b}{2} = \frac{10 + (-12)}{2} = \frac{-2}{2}$$

$\therefore M = -1$

Clave C

Nivel 2 (página 65) Unidad 3

Comunicación matemática

13.

14. I. $1 + \tan^2x = A\sec^2x$

$\Rightarrow A = 1$

II. $\text{sen}^4x - \cos^4x = 1 - B\cos^2x$

$\text{sen}^2x - \cos^2x = 1 - B\cos^2x$

$\Rightarrow B = 2$

III. $(1 - \text{sen}x - D\cos x)^2 = 2(C - \text{sen}x)(1 - \cos x)$

$(1 - \text{sen}x - 1\cos x)^2 = 2(1 - \text{sen}x)(1 - \cos x)$

$\Rightarrow D = 1 \wedge C = 1$

IV. $\text{sen}^4x + \cos^4x = 1 + E\text{sen}^2x\cos^2x$

$\text{sen}^4x + \cos^4x = 1 + (-2)\text{sen}^2x\cos^2x$

$\Rightarrow E = -2$

Hallamos la suma:

$A + B + C + D + E$

$1 + 2 + 1 + 1 + (-2) = 3$

Clave E

Razonamiento y demostración

15. $A = \sec^2x + \dfrac{1 - \tan^2x}{1 - \cot^2x}$

$A = \sec^2x + \dfrac{1 - \dfrac{\text{sen}^2x}{\cos^2x}}{1 - \dfrac{\cos^2x}{\text{sen}^2x}}$

$A = \sec^2x + \dfrac{\dfrac{\cos^2x - \text{sen}^2x}{\cos^2x}}{\dfrac{\text{sen}^2x - \cos^2x}{\text{sen}^2x}}$

$A = \sec^2x + \dfrac{(-)(\text{sen}^2x - \cos^2x)\text{sen}^2x}{\cos^2x(\text{sen}^2x - \cos^2x)}$

$\Rightarrow A = \sec^2x - \tan^2\alpha$

$\therefore A = 1$

Clave A

16. $M = \cot^2\alpha\text{sen}^2\alpha + \tan^2\alpha\cos^2\alpha$

$\Rightarrow M = \dfrac{\cos^2\alpha}{\text{sen}^2\alpha}\text{sen}^2\alpha + \dfrac{\text{sen}^2\alpha}{\cos^2\alpha}\cos^2\alpha$

$\therefore M = \cos^2\alpha + \text{sen}^2\alpha = 1$

Clave B

17. $E = \dfrac{1 - \tan^4x}{1 - \tan^2x}$

$E = \dfrac{(1 + \tan^2x)(1 - \tan^2x)}{1 - \tan^2x}$

Propiedad:

$\sec^2x - \tan^2x = 1$

$\sec^2x = 1 + \tan^2x$

$E = 1 + \tan^2x$

$E = \sec^2x$

Clave E

18. $A = \sqrt{\dfrac{1 + \cos x}{1 - \cos x}} - \csc x$

Propiedad: $\sqrt{\dfrac{1 + \cos x}{1 - \cos x}} = \cot\dfrac{x}{2}$

$A = \cot\dfrac{x}{2} - \csc x$

Propiedad: $\cot\dfrac{x}{2} = \csc x + \cot x$

$A = (\csc x + \cot x) - \csc x$

$\therefore A = \cot x$

Clave D

19. $T = \dfrac{(1 - \cos\alpha)(1 + \sec\alpha)}{\tan\alpha}$

$T = \dfrac{(1 - \cos\alpha)\left(1 + \dfrac{1}{\cos\alpha}\right)}{\dfrac{\text{sen}\alpha}{\cos\alpha}}$

$T = \dfrac{(1 - \cos\alpha)(1 + \cos\alpha)}{\cos\alpha\ .\ \text{sen}\alpha}\cos\alpha$

$T = \dfrac{1 - \cos^2\alpha}{\text{sen}\alpha} = \dfrac{\text{sen}^2\alpha}{\text{sen}\alpha}$

$\therefore T = \text{sen}\alpha$

Clave B

20. $\cos x + \tan x = 1$

Multiplicando por cscx:

$\csc x\cos x + \csc x\tan x = \csc x$

$\dfrac{\cos x}{\text{sen}x} + \csc x\dfrac{\text{sen}x}{\cos x} = \csc x$

$\cot x + \dfrac{1}{\cos x} = \csc x$

$\dfrac{1}{\cos x} = \csc x - \cot x$

$\Rightarrow \csc x + \cot x = \cos x$ (propiedad)

Esta propiedad se deriva de la siguiente identidad:

$\csc^2x = 1 + \cot^2x$

$\csc^2x - \cot^2x = 1$

$(\csc x + \cot x)(\csc x - \cot x) = 1$

$\csc x + \cot x = \dfrac{1}{\csc x - \cot x}$

$\csc x - \cot x = \dfrac{1}{\csc x + \cot x}$

Reemplazando en la expresión R:

$R = \csc x + \cot x + \tan x\csc x$

$R = \cos x + \tan x = 1$ (dato)

$\therefore R = 1$

Clave E

21. $L = [(1 - \cos^2\theta)\cot\theta + \text{sen}\theta\cos\theta]\cot\theta$

$L = [\text{sen}^2\theta\cot\theta + \text{sen}\theta\cos\theta]\cot\theta$

$L = [\text{sen}^2\theta\dfrac{\cos\theta}{\text{sen}\theta} + \text{sen}\theta\cos\theta]\cot\theta$

$L = [\text{sen}\theta\cos\theta + \text{sen}\theta\cos\theta]\cot\theta$

$L = [2\text{sen}\theta\cos\theta]\dfrac{\cos\theta}{\text{sen}\theta} = 2\cos^2\theta$

Clave E

22. $(\tan\theta - \cot\theta)^2 = 3^2$

$\tan^2\theta + \cot^2\theta - \underbrace{2\tan\theta\cot\theta}_{1} = 9$

$\tan^2\theta + \cot^2\theta - 2 = 9$

$\tan^2\theta + \cot^2\theta = 11$

$\Rightarrow C = 11$

Clave E

Resolución de problemas

23. $\left.\begin{array}{l} a\text{sen}x = b\sec z \\ c\cot y = d\text{sen}x \\ e\tan y = f\cos z \end{array}\right\} \times$

$a\ .\ c\ .\ e\ .\ \text{sen}x\cot y\tan y = b.d.f.\sec z\text{sen}x\cos z$

$\Rightarrow a\ .\ c\ .\ e = b\ .\ d\ .\ f$

Clave D

24. ▪ Desarrollamos: $\text{sen}^6x - \cos^6x$

$\text{sen}^6x - \cos^6x = (\text{sen}^2x - \cos^2x)$
$(\text{sen}^4x + \text{sen}^2x\cos^2x + \cos^4x)$

$\text{sen}^6x - \cos^6x = (\text{sen}^2x + \cos^2x - 2\cos^2x)$
$(\text{sen}^4x + \cos^4x + \text{sen}^2x\cos^2x)$

$\text{sen}^6x - \cos^6x = (1 - 2\cos^2x)$
$(1 - 2\text{sen}^2x\cos^2x + \text{sen}^2x\cos^2x)$

$\text{sen}^6x - \cos^6x = (1 - 2\cos^2x)$
$(1 - \text{sen}2x\cos2x)$

$= (1 - A\cos^2x)(1 - B\text{sen}^2x\cos^2x)$

$= (1 - 2\cos^2x)(1 - 1\text{sen}^2x\cos^2x)$

$\therefore A = 2 \wedge B = 1$

$\Rightarrow A + B = 2 + 1 = 3$

Clave B

Nivel 3 (página 66) Unidad 3

Comunicación matemática

25. I. $(\sec\alpha - \tan\alpha)(\sec\alpha - \tan\alpha) = 1$

$\sec^2\alpha - \tan^2\alpha = 1$ (V)

II. $2\text{sen}^2\alpha - 1 = (\text{sen}\alpha + \cos\alpha)(\text{sen}\alpha - \cos\alpha)$

$2\text{sen}^2\alpha - 1 = \text{sen}^2\alpha - \cos^2\alpha$ (V)

III. $1 - 2\cos^2\alpha = 2\text{sen}^2\alpha - 1$

$2 = 2\text{sen}^2\alpha + \cos^2\alpha$ (V)

IV. $\text{sen}^4x - \cos^4x = 1 - \cos^2x$

$\text{sen}^2x - \cos^2x = 1 - \cos^2x$ (F)

$\therefore$ VVVF

Clave C

26. (M) $\text{sen}^3\theta\cos\theta + \text{sen}\theta\cos^3\theta = \dfrac{1}{2}\tan\theta$

$\text{sen}\theta\cos\theta(\text{sen}^2\theta + \cos^2\theta) = \dfrac{\text{sen}\theta}{2\cos\theta}$

$\cos^2\theta = 1/2$

$\cos\theta = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

(N) $\dfrac{(\text{sen}\alpha + \cos\alpha)^2 - 1}{\cos\alpha} = \sqrt{3}$

$\dfrac{\text{sen}^2\alpha + 2\text{sen}\alpha\cos\alpha + \cos^2\alpha - 1}{\cos\alpha} = \sqrt{3}$

$\dfrac{2\text{sen}\alpha\cos\alpha}{\cos\alpha} = \sqrt{3}$

$\text{sen}\alpha = \dfrac{\sqrt{3}}{2} \Rightarrow \alpha = 60°$

$\therefore 45° < 60° \Rightarrow M < N$

Clave C

Razonamiento y demostración

27. $H = \dfrac{\text{sen}^4x - \cos^4x}{\text{sen}^2x - \cos^2x}$

$H = \dfrac{(\text{sen}^2x + \cos^2x)(\text{sen}^2x - \cos^2x)}{\text{sen}^2x - \cos^2x}$

$H = \text{sen}^2x + \cos^2x \qquad \therefore\ H = 1$

Clave B

28. $E = \dfrac{\cos\alpha + \tan\alpha}{\text{sen}\alpha\,\cos\alpha} - \sec^2\alpha$

$E = \dfrac{\cos\alpha + \dfrac{\text{sen}\alpha}{\cos\alpha}}{\text{sen}\alpha\,\cos\alpha} - \sec^2\alpha$

$E = \dfrac{\cos^2\alpha + \text{sen}\alpha}{\text{sen}\alpha\,\cos^2\alpha} - \dfrac{1}{\cos^2\alpha}$

$\Rightarrow\ E = \dfrac{\cos^2\alpha + \text{sen}\alpha - \text{sen}\alpha}{\text{sen}\alpha\,\cos^2\alpha} = \dfrac{1}{\text{sen}\alpha}$

$\therefore\quad E = \csc\alpha$

Clave C

29. $(\text{sen}\alpha + \cos\alpha)^2 = \left(\dfrac{2}{3}\right)^2$

$\text{sen}^2\alpha + 2\text{sen}\alpha\cos\alpha + \cos^2\alpha = \dfrac{4}{9}$

$1 + 2\text{sen}\alpha\cos\alpha = \dfrac{4}{9}$

$\Rightarrow \text{sen}\alpha\cos\alpha = -\dfrac{5}{18}$

Piden:

$E = -18\text{sen}\alpha\cos\alpha$

$E = -18 \cdot \dfrac{(-5)}{18} = 5$

Clave E

30. $(\text{sen}x + \cos x)^2 = A + B\cos x\,\text{sen}x$

$(\text{sen}^2x + \cos^2x + 2\text{sen}x\cos x) = A + B\text{sen}x\cos x$

$1 + 2\text{sen}x\cos x = A + B\text{sen}x\cos x$

$\Rightarrow\ A = 1 \quad \wedge \quad B = 2$

$\therefore\ A + B = 1 + 2 = 3$

Clave E

31. Dato:

$\sec\theta + \tan\theta = 4 \quad ...(I)$

Se sabe:

$\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\sec\theta - \tan\theta = \dfrac{1}{4} \quad ...(II)$

De (I) y (II):

$2\sec\theta = 4 + \dfrac{1}{4} = \dfrac{17}{4}$

$\sec\theta = \dfrac{17}{8} \Rightarrow \cos\theta = \dfrac{8}{17}$

En (I):

$\sec\theta + \tan\theta = 4$

$\dfrac{17}{8} + \tan\theta = 4$

$\tan\theta = 4 - \dfrac{17}{8} = \dfrac{15}{8}$

$\cot\theta = \dfrac{8}{15}$

Piden:

$M = 15\cot\theta + 17\cos\theta$

$M = 15\left(\dfrac{8}{15}\right) + 17\left(\dfrac{8}{17}\right)$

$\therefore\ M = 8 + 8 = 16$

Clave C

32. $(\text{sen}\theta = a)^2 \ \wedge\ (\cos\theta = b)^2$

$\text{sen}^2\theta = a^2 \ \wedge\ \cos^2\theta = b^2$

$\text{sen}^2\theta + \cos^2\theta = a^2 + b^2$

$1 = a^2 + b^2$

Clave C

33. $(\tan\theta + \cot\theta)^2 = (\sqrt{7})^2$

$\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 7$

$\sec^2\theta - 1 + \cot^2\theta + 2 = 7$

$\sec^2\theta + \cot^2\theta + 1 = 7$

$\sec^2\theta + \cot^2\theta = 6$

Clave C

34. $\dfrac{1}{\cos^2x} + \dfrac{1}{\tan^2x} = \dfrac{1}{C} + \dfrac{1}{\cot^2x}$

$\sec^2x + \cot^2x = \dfrac{1}{C} + \tan^2x$

$\sec^2x - \tan^2x + \cot^2x = \dfrac{1}{C}$

$1 + \cot^2x = \dfrac{1}{C}$

$\csc^2x = \dfrac{1}{C} \Rightarrow C = \text{sen}^2x$

Clave B

Resolución de problemas

35. Operamos la primera expresión:

$\left(\dfrac{1 - \text{sen}x\cos x}{1 - \cot x}\right)\left(\dfrac{\text{sen}^4x - \cos^4x}{\text{sen}^3x + \cos^3x}\right) = A\text{sen}x$

$= \left(\dfrac{1 - \text{sen}x\cos x}{1 - \dfrac{\cos x}{\text{sen}x}}\right)$

$\left(\dfrac{(\text{sen}^2x - \cos^2x)(\text{sen}^2x + \cos^2x)}{(\text{sen}x + \cos x)(\text{sen}^2x - \text{sen}x\cos x + \cos^2x)}\right)$

$= \left(\dfrac{1 - \text{sen}x\cos x}{\dfrac{\text{sen}x - \cos x}{\text{sen}x}}\right)$

$\left(\dfrac{(\text{sen}x - \cos x)(\text{sen}x + \cos x)}{(\text{sen}x + \cos x)(1 - \text{sen}x\cos x)}\right)$

$\text{sen}x = A\text{sen}x$

$\therefore\ A = 1$

En la siguiente expresión tenemos:

$\left(\dfrac{1 + \text{sen}x + \cos x}{\sqrt{B}}\right)^2 = (1 + \text{sen}x)(1 + \cos x)$

$\dfrac{(1 + \text{sen}x + \cos x)^2}{B} = (1 + \text{sen}x)(1 + \cos x)$

$(1 + \text{sen}x + \cos x)^2 = B(1 + \text{sen}x)(1 + \cos x)$

$\therefore\ B = 2$

$\Rightarrow\ A = 1 \ \wedge\ B = 2$

Clave D

36. $2 = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + ...}}}$

$2 = \sqrt{\tan x + 2}$

$4 = \tan x + 2 \ \Rightarrow\ \tan x = 2$

Desarrollamos E:

$E = \dfrac{\sec x\csc x - \cot x}{\text{sen}x}$

$E = \dfrac{\dfrac{1}{\cos x\,\text{sen}x} - \dfrac{\cos x}{\text{sen}x}}{\text{sen}x}$

$E = \dfrac{\dfrac{1 - \cos^2x}{\text{sen}x\cos x}}{\text{sen}x} = \dfrac{\text{sen}^2x}{\text{sen}^2x\cos x}$

$E = \sec x$

$E^2 = \sec^2x = \tan^2x + 1$

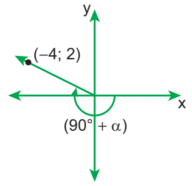
$E^2 = (2)^2 + 1 = 5 \ \Rightarrow\ E^2 = 5$

$\therefore\ E = \sqrt{5}$

Clave A

MARATÓN MATEMÁTICA (página 67)

1.



Del gráfico:

$\cot(90° + \alpha) = \dfrac{x}{y} = \dfrac{-4}{2}$

$\cot(90° + \alpha) = -2$

$-\tan\alpha = -2$

$\tan\alpha = 2$

Nos piden:

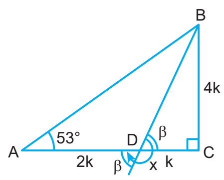
$k = \sqrt{\sec^2\alpha - 1}$

$k = \sqrt{\tan^2\alpha} = \tan\alpha$

$\therefore k = \tan\alpha = 2$

Clave D

2.



$x = (180° + \beta)$

$\Rightarrow \tan x = \tan(180° + \beta)$

$\tan x = \tan(\beta)$

$\tan x = \dfrac{4k}{k} = 4$

$\therefore \tan x = 4$

Clave D

3. Por identidades trigonométricas, tenemos:

$A = \dfrac{\cos^4 x}{1 + \tan^2 x} + \dfrac{\text{sen}^4 x}{1 + \cot^2 x} + 3\text{sen}^2 x\cos^2 x$

$A = \dfrac{\cos^4 x}{\sec^2 x} + \dfrac{\text{sen}^4 x}{\csc^2 x} + 3\text{sen}^2 x\cos^2 x$

$A = \cos^6 x + \text{sen}^6 x + 3\text{sen}^2 x\cos^2 x$

$A = 1 - 3\text{sen}^2 x\cos^2 x + 3\text{sen}^2 x\cos^2 x$

$\therefore A = 1$

Clave E

4. Sean a y b los ángulos coterminales, además $\alpha > \beta$, según el enunciado, planteamos:

$\dfrac{\alpha}{\beta} = \dfrac{5}{2} \Rightarrow \alpha = 5k \wedge \beta = 2k$

$1050° < 5k + 2k < 1800°$

$1050° < 7k < 1800°$

$150° < k < 257°, 1...$ ... (1)

Como α y β son ángulos coterminales, entonces:

$\alpha - \beta = 360°\, n, (n \in \mathbb{Z})$

$3k = 360°\, n \Rightarrow k = 120°\, n$ ... (2)

Reemplazamos (2) en (1):

$150° < 120°\, n < 257°, 1...$

$1{,}25 < n < 2{,}14 \Rightarrow n = 2$

$k = 120°\, n$

$k = 240°$

$\therefore \alpha = 5k = 5(240°) \Rightarrow \alpha = 1200°$

Clave A

5. Tengamos presente: $\sec^2\alpha = 1 + \tan^2\alpha$

Luego:

$P = (\sec^2\alpha - \tan^2\alpha)(\sec^2\alpha - \tan^2\alpha + 2\tan^2\alpha)(1 + 2(\tan^2\alpha + 1)\tan^2\alpha) + \tan^8\alpha$

$P = (\sec^2\alpha - \tan^2\alpha)(\sec^2\alpha + \tan^2\alpha)(1 + 2\tan^2\alpha + \tan^4\alpha + \tan^4\alpha) + \tan^8\alpha$

$P = (\sec^4\alpha - \tan^4\alpha)((\tan^2\alpha + 1)^2 + \tan^4\alpha) + \tan^8\alpha$

$P = (\sec^4\alpha - \tan^4\alpha)(\sec^4\alpha + \tan^4\alpha) + \tan^8\alpha$

$P = \sec^8\alpha - \tan^8\alpha + \tan^8\alpha = \sec^8\alpha$

$\therefore P = \sec^8\alpha$

Clave D

6. Tenemos: $\cos^2 x = 1 - \text{sen}^2 x$

$\Rightarrow 2\text{sen}^2 x = 4(1 - \text{sen}^2 x) - 5\text{sen}x$

$6\text{sen}^2 x + 5\text{sen}x - 4 = 0$

$(2\text{sen}x - 1)(3\text{sen}x + 4) = 0$

$\Rightarrow \text{sen}x = \dfrac{1}{2} \wedge \text{sen}x = -\dfrac{4}{3}$

$\text{sen}x \in [-1; 1]$

$\therefore \text{sen}x = \dfrac{1}{2}$

Clave B

7. $R = \left(\dfrac{\cos x}{1 - \text{sen}x} \cdot \dfrac{(1 + \text{sen}x)}{(1 + \text{sen}x)} - \dfrac{1}{\cot x}\right)\left(\dfrac{\text{sen}x}{1 + \cos x} \cdot \dfrac{1 - \cos x}{1 - \cos x} - \dfrac{1}{\tan x}\right)$

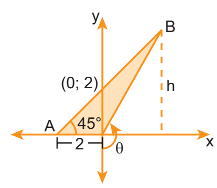
$R = \left(\dfrac{1 + \text{sen}x}{\cos x} - \tan x\right)\left(\dfrac{1 - \cos x}{\text{sen}x} + \cot x\right)$

$R = \left(\dfrac{1}{\cos x} + \dfrac{\text{sen}x}{\cos x} - \tan x\right)\left(\dfrac{1}{\text{sen}x} - \dfrac{\cos x}{\text{sen}x} + \cot x\right)$

$R = \sec x \csc x$

Clave D

8.



$A_{\Delta AOB} = 4u^2 = \dfrac{2h}{2} \Rightarrow h = 4\ m$

El punto: $B = (2; 4)$

$\Rightarrow \tan\theta = \dfrac{-2}{4} = \dfrac{-1}{2}$

Clave A

9. $M = \dfrac{\cos^2 b}{\cos b} + \dfrac{\text{sen}^2 b}{\cos b} + \dfrac{\text{sen}b}{\cos b} - \dfrac{1}{\cos b}$

$M = \dfrac{\cos^2 b + \text{sen}^2 b + \text{sen}b - 1}{\cos b}$

$\therefore M = \dfrac{\text{son}b}{\cos b} = \tan b$

Clave E

Unidad 4

ÁNGULOS COMPUESTOS

APLICAMOS LO APRENDIDO (página 70) Unidad 4

1. $M = \text{sen}27°\cos10° + \cos27°\text{sen}10°$

$M = \text{sen}(27° + 10°)$

$M = \text{sen}37° = \frac{3}{5}$

$\therefore M = \frac{3}{5}$

Clave D

2. $R = \dfrac{\tan 20° + \tan 17°}{1 - \tan 20° \tan 17°}$

$R = \tan(20° + 17°)$

$R = \tan37° = \frac{3}{4}$

$\therefore R = \frac{3}{4}$

Clave E

3. $M = \dfrac{\text{sen}(x+y)}{\cos x.\cos y} - \tan y$

$M = \dfrac{\text{sen}x\cos y + \cos x\text{sen}y}{\cos x \cos y} - \tan y$

$M = \dfrac{\text{sen}x\cos y}{\cos x\cos y} + \dfrac{\cos x\text{sen}y}{\cos x\cos y} - \tan y$

$M = \tan x + \tan y - \tan y$

$\therefore M = \tan x$

Clave A

4. $A = \dfrac{\text{sen}(x+y) - \text{sen}(x-y)}{\cos(x+y) + \cos(x-y)}$

$A = \dfrac{(\text{sen}x\cos y + \cos x\text{sen}y) - (\text{sen}x\cos y - \cos x\text{sen}y)}{(\cos x\cos y - \text{sen}x\text{sen}y) + (\cos x\cos y + \text{sen}x\text{sen}y)}$

$A = \dfrac{2\cos x\text{sen}y}{2\cos x\cos y} = \dfrac{\text{sen}y}{\cos y} = \tan y$

$\therefore A = \tan y$

Clave E

5. $A = \sqrt{2}\cos(\alpha + 45°) + \text{sen}\alpha$

$A = \sqrt{2}(\cos\alpha\cos45° - \text{sen}\alpha\text{sen}45°) + \text{sen}\alpha$

$A = \sqrt{2}\left(\cos\alpha.\frac{\sqrt{2}}{2} - \text{sen}\alpha.\frac{\sqrt{2}}{2}\right) + \text{sen}\alpha$

$A = \cos\alpha - \text{sen}\alpha + \text{sen}\alpha = \cos\alpha$

$\therefore A = \cos\alpha$

Clave E

6. Por dato:

$\tan\alpha = 1 \quad \wedge \quad \tan\theta = \frac{3}{4}$

Piden:

$S = 28\tan(\alpha - \theta)$

$S = 28\left(\dfrac{\tan\alpha - \tan\theta}{1 + \tan\alpha\tan\theta}\right)$

$S = 28\left(\dfrac{1 - \frac{3}{4}}{1 + 1.\frac{3}{4}}\right) = 28\left(\dfrac{\frac{1}{4}}{\frac{7}{4}}\right) = 28.\frac{1}{7}$

$\therefore S = 4$

Clave A

7. $M = (\cos x + \cos y)^2 + (\text{sen}x + \text{sen}y)^2$

Entonces:

$(\cos x + \cos y)^2 = \cos^2x + 2\cos x\cos y + \cos^2y$

$(\text{sen}x + \text{sen}y)^2 = \text{sen}^2x + 2\text{sen}x\text{sen}y + \text{sen}^2y \quad (+)$

$M = 1 + 2(\cos x\cos y + \text{sen}x\text{sen}y) + 1$

$M = 2 + 2\cos(x - y)$

Por dato: $x - y = 60°$

Luego:

$M = 2 + 2\cos60° = 2 + 2\left(\frac{1}{2}\right)$

$\therefore M = 3$

Clave C

8. $R = \text{sen}(x + y)\text{sen}(x - y) + \text{sen}^2y$

$R = (\text{sen}x\cos y + \cos x\text{sen}y)(\text{sen}x\cos y - \cos x\text{sen}y) + \text{sen}^2y$

$R = (\text{sen}x\cos y)^2 - (\cos x\text{sen}y)^2 + \text{sen}^2y$

$R = \text{sen}^2x\cos^2y - \cos^2x\text{sen}^2y + \text{sen}^2y$

$R = \text{sen}^2x\cos^2y + \text{sen}^2y(1 - \cos^2x)$

$R = \text{sen}^2x\cos^2y + \text{sen}^2y\text{sen}^2x$

$R = \text{sen}^2x\underbrace{(\cos^2y + \text{sen}^2y)}_{1} = \text{sen}^2x$

$\therefore R = \text{sen}^2x$

Clave A

9. $N = \dfrac{2\cos(\theta - 30°) - \sqrt{3}\cos\theta}{\text{sen}\theta}$

$N = \dfrac{2(\cos\theta\cos30° + \text{sen}\theta\text{sen}30°) - \sqrt{3}\cos\theta}{\text{sen}\theta}$

$N = \dfrac{2\left[\cos\theta\left(\frac{\sqrt{3}}{2}\right) + \text{sen}\theta\left(\frac{1}{2}\right)\right] - \sqrt{3}\cos\theta}{\text{sen}\theta}$

$N = \dfrac{\sqrt{3}\cos\theta + \text{sen}\theta - \sqrt{3}\cos\theta}{\text{sen}\theta} = \dfrac{\text{sen}\theta}{\text{sen}\theta} = 1$

$\therefore N = 1$

Clave C

10. Por dato:

$A + B + C = \pi = 180°$

Además: $\tan B + \tan C = 2\tan A$

Piden: $\cot B . \cot C$

Por propiedad:

$\tan A + \underbrace{\tan B + \tan C}_{2\tan A} = \tan A . \tan B . \tan C$

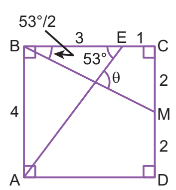
$3\tan A = \tan A . \tan B . \tan C$

$3 = \tan B . \tan C$

$\Rightarrow \cot B . \cot C = \frac{1}{3}$

Clave D

11.



$\triangle$ ABE: (37° y 53°)

$m\angle BEA = 53°$

$\triangle$ BCM: $m\angle CBM = 53°/2$

Luego: $\theta = 53° + 53°/2$

$\tan\theta = \tan(53° + 53°/2)$

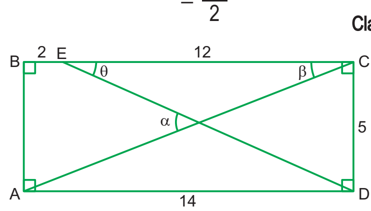
$= \dfrac{\tan 53° + \tan 53°/2}{1 - \tan 53°.\tan 53°/2}$

$= \dfrac{\frac{4}{3} + \frac{1}{2}}{1 - \frac{4}{3}.\frac{1}{2}}$

$= \frac{11}{2}$

Clave C

12.



$\triangle$ ECD: $ED = \sqrt{12^2 + 5^2}$

$= \sqrt{169} = 13$

$\triangle$ ABC: $AC = \sqrt{14^2 + 5^2}$

$= \sqrt{221}$

Luego:

$\alpha = \theta + \beta$

$\sqrt{221}\,\text{sen}\alpha = \sqrt{221}\,\text{sen}(\theta + \beta)$

$\sqrt{221}\,\text{sen}\alpha = \sqrt{221}\,(\text{sen}\theta\cos\beta + \cos\theta\text{sen}\beta)$

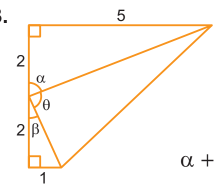
$= \sqrt{221}\left(\frac{5}{13}.\frac{14}{\sqrt{221}} + \frac{12}{13}.\frac{5}{\sqrt{221}}\right)$

$= \frac{70}{13} + \frac{60}{13}$

$\sqrt{221}\,\text{sen}\alpha = 10$

Clave D

13.



$\alpha + \beta + \theta = 180°$

Entonces: $\tan\alpha + \tan\theta + \tan\beta = \tan\alpha.\tan\theta.\tan\beta$

$\frac{5}{2} + \tan\theta + \frac{1}{2} = \frac{5}{2}.\tan\theta.\frac{1}{2}$

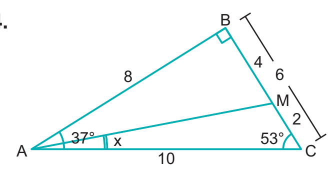
$3 + \tan\theta = \frac{5}{4}\tan\theta$

$3 = \frac{1}{4}\tan\theta$

$\tan\theta = 12$

Clave A

14.



$\tan(37° - x) = \frac{4}{8}$

$\dfrac{\tan 37° - \tan x}{1 + \tan 37°.\tan x} = \frac{1}{2}$

$\dfrac{\frac{3}{4} - \tan x}{1 + \frac{3}{4}.\tan x} = \frac{1}{2}$

$\frac{3}{2} - 2\tan x = 1 + \frac{3}{4}\tan x$

$\frac{1}{2} = \frac{11}{4}\tan x$

$\tan x = \frac{2}{11}$

Clave B

PRACTIQUEMOS

Nivel 1 (página 72) Unidad 4

Comunicación matemática

1.

2.

Razonamiento y demostración

3. $T = \text{sen}8°\cos22° + \cos8°\text{sen}22°$

Por identidad:

$\text{sen}(x + y) = \text{sen}x\cos y + \cos x\text{sen}y$

$\Rightarrow T = \text{sen}(8° + 22°) = \text{sen}(30°) = \frac{1}{2}$

Clave B

4. $I = \text{sen}4° \,.\, \cos 2° - \cos 4° \,.\, \text{sen}2°$

Por propiedad:

$\text{sen}(x - y) = \text{sen}x\cos y - \cos x\,\text{sen}y$

$\Rightarrow I = \text{sen}(4° - 2°) = \text{sen}2°$

Clave C

5. $M = \cos 40°\cos 13° - \text{sen}40°\text{sen}13°$

Por identidad:

$\cos(x + y) = \cos x\cos y - \text{sen}x\,\text{sen}y$

$\Rightarrow M = \cos(40° + 13°) = \cos 53° = \frac{3}{5}$

Clave E

6. $R = \cos 80°\cos 50° + \text{sen}80°\text{sen}50°$

Por identidad:

$\cos(x - y) = \cos x\cos y + \text{sen}x\,\text{sen}y$

$\Rightarrow R = \cos(80° - 50°) = \cos 30° = \frac{\sqrt{3}}{2}$

Clave D

7. $A = \dfrac{\tan 70° - \tan 10°}{1 + \tan 70° \tan 10°}$

Por identidad:

$\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

$\Rightarrow A = \tan(70° - 10°) = \tan 60° = \sqrt{3}$

Clave A

8. $A = \dfrac{\text{sen}(x+y)}{\cos x\cos y} - \tan y + \tan x$

$A = \dfrac{\text{sen}x\cos y + \cos x\,\text{sen}y}{\cos x\cos y} - \dfrac{\text{sen}y}{\cos y} + \tan x$

$A = \dfrac{\text{sen}x\cos y + \cos x\,\text{sen}y - \text{sen}y\cos x}{\cos x\cos y} + \tan x$

$A = \dfrac{\text{sen}x\cos y}{\cos x\cos y} + \tan x = \dfrac{\text{sen}x}{\cos x} + \tan x = 2\tan x$

Clave D

9. $\tan\alpha = \frac{3}{4}$; $\tan\theta = \frac{5}{12}$

$\tan(\alpha + \theta) = \dfrac{\tan\theta + \tan\alpha}{1 - \tan\alpha\,.\,\tan\theta}$

$\tan(\alpha + \theta) = \dfrac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4}\,.\,\frac{5}{12}} = \dfrac{\frac{9+5}{12}}{1 - \frac{5}{16}}$

$\tan(\alpha + \theta) = \dfrac{\frac{14}{12}}{\frac{11}{16}} = \dfrac{14\,.\,16}{12\,.\,11} = \dfrac{56}{33}$

Clave C

10. $\tan\alpha = \frac{3}{4}$ $\tan\beta = \frac{1}{4}$

$\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\tan(\alpha - \beta) = \dfrac{\frac{3}{4} - \frac{1}{4}}{1 + \frac{3}{4}\,.\,\frac{1}{4}} = \dfrac{\frac{2}{4}}{1 + \frac{3}{16}} = \dfrac{\frac{1}{2}}{\frac{19}{16}}$

$\tan(\alpha - \beta) = \dfrac{16}{2.\,19} = \dfrac{8}{19}$

Clave E

Nivel 2 (página 72) Unidad 4

Comunicación matemática

11.

12.

Razonamiento y demostración

13. $M = (\text{sen}(x + y) - \cos x\,\text{sen}y)\,.\,\sec y$

$M = (\text{sen}x\cos y + \text{sen}y\cos x - \cos x\,\text{sen}y)\sec y$

$M = (\text{sen}x\cos y)\,.\,\dfrac{1}{\cos y} = \text{sen}x$

Clave B

14. $A = [\cos(x + y) + \text{sen}x\,\text{sen}y]\sec x$

$A = (\cos x\cos y - \text{sen}x\,\text{sen}y + \text{sen}x\,\text{sen}y)\,.\,\dfrac{1}{\cos x}$

$A = (\cos x\cos y)\,.\,\dfrac{1}{\cos x} = \cos y$

Clave E

15. $S = \dfrac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$

Por identidad:

$\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

$\Rightarrow S = \tan(2x + 3x) = \tan 5x$

Clave C

16. $T = 2\text{sen}(x + 30°) - \sqrt{3}\,\text{sen}x$

$T = 2\text{sen}x\cos 30° + 2\cos x\,\text{sen}30° - \sqrt{3}\,\text{sen}x$

$T = 2\text{sen}x\,.\,\frac{\sqrt{3}}{2} + 2\cos x\,.\,\frac{1}{2} - \sqrt{3}\,\text{sen}x$

$T = \sqrt{3}\,\text{sen}x + \cos x - \sqrt{3}\,\text{sen}x = \cos x$

Clave A

17. $\text{sen}x\cos 21° + \cos x\,\text{sen}21° = \text{sen}34°$

Identidad: $\text{sen}(a + b) = \text{sen}a\cos b + \cos a\,\text{sen}b$

$\Rightarrow \text{sen}(x + 21°) = \text{sen}34°$

$\Rightarrow x + 21° = 34° \Rightarrow x = 13°$

Clave A

18. $\text{sen}\theta\cos 9° - \cos\theta\,\text{sen}9° = \text{sen}27°$

Identidad: $\text{sen}(\alpha - \beta) = \text{sen}\alpha\cos\beta - \cos\alpha\,\text{sen}\beta$

$\Rightarrow$ $\text{sen}(\theta - 9°) = \text{sen}27°$

$\Rightarrow$ $\theta - 9° = 27° \Rightarrow \theta = 36°$

Clave C

19. $\cos 26°\cos\theta + \text{sen}26°\text{sen}\theta = \cos 19°$

Identidad: $\cos(a - b) = \cos a\cos b + \text{sen}a\,\text{sen}b$

$\Rightarrow$ $\cos(26° - \theta) = \cos(19°)$

$\Rightarrow$ $26° - \theta = 19° \Rightarrow \theta = 7°$

Clave D

20. $\cos 34°\cos x - \text{sen}34°\text{sen}x = \cos 58°$

Identidad: $\cos(a + b) = \cos a\cos b - \text{sen}a\,\text{sen}b$

$\Rightarrow$ $\cos(34° + x) = \cos(58°)$

$\Rightarrow$ $34° + x = 58° \Rightarrow x = 24°$

Clave E

Nivel 3 (página 73) Unidad 4

Comunicación matemática

21.

22.

Razonamiento y demostración

23. $Z = \dfrac{\cos(a + b)}{\text{sen}a\,.\,\cos b} + \tan b$

$Z = \dfrac{\cos a\cos b - \text{sen}a\,\text{sen}b}{\text{sen}a\,.\,\cos b} + \dfrac{\text{sen}b}{\cos b}$

$Z = \dfrac{\cos a\cos b - \text{sen}a\,\text{sen}b + \text{sen}a\,.\,\text{sen}b}{\text{sen}a\,.\,\cos b}$

$Z = \dfrac{\cos a\,.\,\cos b}{\text{sen}a\,.\,\cos b} = \dfrac{\cos a}{\text{sen}a} = \cot a$

Clave A

24. $N = \dfrac{\cos(\alpha - \theta)}{\text{sen}\alpha\,.\,\cos\theta} - \tan\theta$

$N = \dfrac{\cos\alpha\cos\theta + \text{sen}\alpha\,\text{sen}\theta}{\text{sen}\alpha\,.\,\cos\theta} - \dfrac{\text{sen}\theta}{\cos\theta}$

$N = \dfrac{\cos\alpha\cos\theta + \text{sen}\alpha\,\text{sen}\theta - \text{sen}\alpha\,\text{sen}\theta}{\text{sen}\alpha\cos\theta}$

$N = \dfrac{\cos\alpha\,.\,\cos\theta}{\text{sen}\alpha\,.\,\cos\theta} = \dfrac{\cos\alpha}{\text{sen}\alpha} = \cot\alpha$

Clave D

25. $A = (\cos(x + y) + \cos(x - y))\dfrac{\tan y}{2}$

$A = (\cos x\cos y - \text{sen}x\,\text{sen}y + \cos x\cos y + \text{sen}x\,\text{sen}y)\,.\,\dfrac{\text{sen}y}{2\cos y}$

$A = (2\cos x\cos y)\,.\,\dfrac{\text{sen}y}{2\cos y} = \text{sen}y\cos x$

Clave C

26. $\text{sen}(45° + \alpha) = n(\text{sen}\alpha + \cos\alpha)$

$\text{sen}45°\cos\alpha + \cos 45°\text{sen}\alpha = n(\text{sen}\alpha + \cos\alpha)$

$\frac{\sqrt{2}}{2}\cos\alpha + \frac{\sqrt{2}}{2}\text{sen}\alpha = n(\text{sen}\alpha + \cos\alpha)$

$\frac{\sqrt{2}}{2}(\text{sen}\alpha + \cos\alpha) = n(\text{sen}\alpha + \cos\alpha)$

$\Rightarrow n = \frac{\sqrt{2}}{2}$

Clave E

27. $\text{sen}(60° - \theta) = A(\sqrt{3}\cos\theta - \text{sen}\theta)$

$\text{sen}60°\cos\theta - \cos 60°\text{sen}\theta = A(\sqrt{3}\cos\theta - \text{sen}\theta)$

$\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\text{sen}\theta = A(\sqrt{3}\cos\theta - \text{sen}\theta)$

$\frac{1}{2}(\sqrt{3}\cos\theta - \text{sen}\theta) = A(\sqrt{3}\cos\theta - \text{sen}\theta)$

$\Rightarrow A = \frac{1}{2}$

Clave B

28. $T = \text{sen}(A + B)\text{sen}(A - B) + \text{sen}^2B$

Por identidad auxiliar de ángulos compuestos.

$\text{sen}(A + B)\text{sen}(A - B) = \text{sen}^2A - \text{sen}^2B$

$\Rightarrow T = \text{sen}^2A - \text{sen}^2B + \text{sen}^2B$

$T = \text{sen}^2A$

Clave B

29. $M = \text{sen}(A + B)\text{sen}(A - B) - \text{sen}^2A + \text{sen}^2B$

Por identidad auxiliar de ángulos compuestos:

$\text{sen}(A + B)\text{sen}(A - B) = \text{sen}^2A - \text{sen}^2B$

$\Rightarrow M = \text{sen}^2A - \text{sen}^2B - \text{sen}^2A + \text{sen}^2B$

$\Rightarrow M = 0$

Clave C

30. $I = \tan 40° + \tan 13° + \tan 40°\tan 13°\tan 53°$

De la identidad: $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

Tenemos:

$\tan(x + y) = \tan x + \tan y + \tan x\tan y\tan(x + y)$

En la expresión:

$I = \tan(40°) + \tan 13° + \tan 40°\tan 13°\tan(40° + 13°)$

$\Rightarrow I = \tan 53° = \frac{4}{3}$

Clave B

APLICAMOS LO APRENDIDO (página 74) Unidad 4

1. $E = \dfrac{1+\cos 20°}{\text{sen}20°}$

$E = \dfrac{\left(2\cos^2 10°\right)}{\text{sen}20°}$

$E = \dfrac{2\cos^2 10°}{2\text{sen}10°\cos 10°} = \dfrac{\cos 10°}{\text{sen}10°} = \cot 10°$

$\therefore E = \cot 10°$

Clave B

2. Del dato:

$\tan x + \cot x = 5$

$\sec x \csc x = 5$

$\Rightarrow \cos x \,\text{sen}x = \dfrac{1}{5}$

Piden: sen2x

$\text{sen}2x = 2\text{sen}x\cos x = 2\left(\dfrac{1}{5}\right) = \dfrac{2}{5}$

$\therefore \text{sen}2x = \dfrac{2}{5}$

Clave B

3.

$\cos 2x + 2\cos x + 1 = 0$

$(2\cos^2 x - 1) + 2\cos x + 1 = 0$

$2\cos^2 x + 2\cos x = 0$

$2\cos x(\cos x + 1) = 0$

$\Rightarrow \cos x = 0 \quad \vee \quad \cos x = -1$

$\therefore x = \dfrac{\pi}{2} \quad \vee \quad x = \pi$

Clave A

4. $E = 8\text{sen}x\cos x\cos 2x\cos 4x$

$E = 4\,.\,\underbrace{2\text{sen}x\cos x}_{\text{sen}2x}\,.\,\cos 2x\cos 4x$

$E = 2\,.\,\underbrace{2\text{sen}2x\cos 2x}_{\text{sen}4x}\,.\,\cos 4x$

$E = 2\text{sen}4x\cos 4x = \text{sen}8x$

$\therefore E = \text{sen}8x$

Clave A

5. $\cos\theta = \dfrac{1}{4} \ \wedge\ \theta \in \left\langle 0;\ \dfrac{\pi}{2}\right\rangle$

$\Rightarrow \dfrac{\theta}{2} \in \left\langle 0;\ \dfrac{\pi}{4}\right\rangle$

Como $\dfrac{\theta}{2} \in \text{IC} \Rightarrow \cos\dfrac{\theta}{2} > 0$

Luego: $\cos\dfrac{\theta}{2} = +\sqrt{\dfrac{1+\cos\theta}{2}}$

$\cos\dfrac{\theta}{2} = \sqrt{\dfrac{1+\left(\dfrac{1}{4}\right)}{2}} = \sqrt{\dfrac{\left(\dfrac{5}{4}\right)}{2}}$

$\cos\dfrac{\theta}{2} = \dfrac{\sqrt{5}}{2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{10}}{4}$

$\therefore \cos\dfrac{\theta}{2} = \dfrac{\sqrt{10}}{4}$

Clave A

6. $\cos\alpha = \dfrac{1}{8} \ \wedge\ \alpha \in \left\langle 2\pi;\ \dfrac{5\pi}{2}\right\rangle$

$\Rightarrow \dfrac{\alpha}{2} \in \left\langle \pi;\ \dfrac{5\pi}{4}\right\rangle$

Como $\dfrac{\alpha}{2} \in \text{IIIC} \Rightarrow \text{sen}\dfrac{\alpha}{2} < 0$

Luego:

$\text{sen}\dfrac{\alpha}{2} = -\sqrt{\dfrac{1-\cos\alpha}{2}}$

$\text{sen}\dfrac{\alpha}{2} = -\sqrt{\dfrac{1-\left(\dfrac{1}{8}\right)}{2}} = -\sqrt{\dfrac{\dfrac{7}{8}}{2}}$

$\text{sen}\dfrac{\alpha}{2} = -\sqrt{\dfrac{7}{16}} = -\dfrac{\sqrt{7}}{4}$

$\therefore \text{sen}\dfrac{\alpha}{2} = -\dfrac{\sqrt{7}}{4}$

Clave C

7. $\cos x = -\dfrac{1}{3} \ \wedge\ x \in \left\langle -\pi;\ -\dfrac{\pi}{2}\right\rangle$

$\Rightarrow \dfrac{x}{2} \in \left\langle -\dfrac{\pi}{2};\ -\dfrac{\pi}{4}\right\rangle$

Como $\dfrac{x}{2} \in \text{IVC} \Rightarrow \tan\dfrac{x}{2} < 0$

Luego:

$\tan\dfrac{x}{2} = -\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

$\tan\dfrac{x}{2} = -\sqrt{\dfrac{1-\left(-\dfrac{1}{3}\right)}{1+\left(-\dfrac{1}{3}\right)}} = -\sqrt{\dfrac{1+\dfrac{1}{3}}{1-\dfrac{1}{3}}}$

$\tan\dfrac{x}{2} = -\sqrt{\dfrac{\dfrac{4}{3}}{\dfrac{2}{3}}} = -\sqrt{\dfrac{4}{2}}$

$\therefore \tan\dfrac{x}{2} = -\sqrt{2}$

Clave C

8. $\cos\dfrac{\pi}{8} = \cos\dfrac{180°}{8} = \cos\dfrac{45°}{2}$

Por identidad:

$\cos\dfrac{x}{2} = \pm\sqrt{\dfrac{1+\cos x}{2}}$

Para: $x = 45°$

$\cos\dfrac{45°}{2} = \pm\sqrt{\dfrac{1+\cos 45°}{2}}$

$\cos\dfrac{45°}{2} = \pm\sqrt{\dfrac{1+\left(\dfrac{\sqrt{2}}{2}\right)}{2}}$

Como $\dfrac{45°}{2} \in \text{IC} \Rightarrow \cos\dfrac{45°}{2} > 0$

$\cos\dfrac{45°}{2} = +\sqrt{\dfrac{\dfrac{2+\sqrt{2}}{2}}{2}}$

$\cos\dfrac{45°}{2} = \sqrt{\dfrac{2+\sqrt{2}}{4}}$

$\therefore \cos\dfrac{\pi}{8} = \dfrac{\sqrt{2+\sqrt{2}}}{2}$

Clave D

9. Piden: $B = \tan 111°$

$B = \tan 3(37°)$

$\tan 3(37°) = \dfrac{3\tan 37° - \tan^3 37°}{1 - 3\tan^2 37°}$

$\tan 111° = \dfrac{3\left(\dfrac{3}{4}\right) - \left(\dfrac{3}{4}\right)^3}{1 - 3\left(\dfrac{3}{4}\right)^2}$

$\tan 111° = \dfrac{\dfrac{9}{4} - \dfrac{27}{64}}{1 - \dfrac{27}{16}} = \dfrac{\dfrac{144-27}{64}}{\dfrac{16-27}{16}}$

$\tan 111° = \dfrac{\dfrac{117}{64}}{\dfrac{-11}{16}} = -\dfrac{117}{44}$

$\therefore \tan 111° = -\dfrac{117}{44}$

Clave C

10. $N = \dfrac{\cos 3x - 4\cos^3 x + 3\cos x - 1}{\cos 2x - 2\cos^2 x + 1 - \text{sen}60°}$

$N = \dfrac{\cos 3x - \overbrace{\left(4\cos^3 x - 3\cos x\right)}^{\cos 3x} - 1}{\cos 2x - \underbrace{\left(2\cos^2 x - 1\right)}_{\cos 2x} - \text{sen}60°}$

$N = \dfrac{\cos 3x - \cos 3x - 1}{\cos 2x - \cos 2x - \text{sen}60°} = \dfrac{-1}{-\text{sen}60°} = \dfrac{1}{\left(\dfrac{\sqrt{3}}{2}\right)}$

$N = \dfrac{2}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

$\therefore N = \dfrac{2\sqrt{3}}{3}$

Clave C

11. $A = \text{sen}12°\text{sen}48°\text{sen}72°$

$A = \text{sen}12°\text{sen}(60° - 12°)\text{sen}(60° + 12°)$

$A = \dfrac{\text{sen}3.(12°)}{4} = \dfrac{\text{sen}36°}{4}$

$\therefore\ A = \dfrac{1}{4}\text{sen}36°$

Clave D

12. $M = \dfrac{\tan 40°.\tan 80°}{\cot 20°}$

$M = \tan 40°\,.\,\tan 80°\,.\,\tan 20°$

$M = \tan 20°\,.\,\tan 40°\,.\,\tan 80°$

$M = \tan 20°\,.\,\tan(60° - 20°)\,.\,\tan(60° + 20°)$

$M = \tan 3\,.\,(20°) = \tan 60° = \sqrt{3}$

$\therefore M = \sqrt{3}$

Clave C

13. $\cot\theta = \dfrac{AB}{1} \quad \Rightarrow \cot\theta = AB$

⊿ ABC: $\tan 2\theta = \dfrac{4}{AB}$

$\Rightarrow \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{4}{\cot\theta}$

$2\tan\theta\cot\theta = 4(1 - \tan^2\theta)$

$\dfrac{1}{2} = 1 - \tan^2\theta$

$\tan\theta = \dfrac{\sqrt{2}}{2}$

Clave D

14. Sabemos que:

$\cos\alpha\cos(60° - \alpha)\cos(60° + \alpha) = \cos 3\alpha$

En la expresión:

$$M = \frac{4\cos 12°.\cos(60° - 12).\cos(60° + 12)}{4}$$

$$M = \frac{\cos 3(12°)}{4}$$

$$M = \frac{\cos 36°}{4}$$

Clave E

PRACTIQUEMOS

Nivel 1 (página 76) Unidad 4

Comunicación matemática

1.
2.

Razonamiento y demostración

3. Como: $\tan\alpha = \frac{2}{3}$

$\Rightarrow \operatorname{sen}\alpha = \frac{2}{\sqrt{13}} \qquad \cos\alpha = \frac{3}{\sqrt{13}}$

$\operatorname{sen}2\alpha = 2\operatorname{sen}\alpha\cos\alpha$

$\operatorname{sen}2\alpha = 2 \cdot \frac{2}{\sqrt{13}} \cdot \frac{3}{\sqrt{13}}$

$\operatorname{sen}2\alpha = \frac{12}{13}$

$C = 13\operatorname{sen}2\alpha + 1 = \frac{13(12)}{13} + 1 = 13$

Clave C

4. Dato: $\cot\theta = \sqrt{7}$

$\Rightarrow \operatorname{sen}\theta = \frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\theta = \frac{\sqrt{7}}{2\sqrt{2}}$

$\cos2\theta = \cos^2\theta - \operatorname{sen}^2\theta$

$\cos2\theta = \left(\frac{\sqrt{7}}{2\sqrt{2}}\right)^2 - \left(\frac{1}{2\sqrt{2}}\right)^2$

$\cos2\theta = \frac{7}{8} - \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$

$L = 4\cos2\theta + 3 = 4\left(\frac{3}{4}\right) + 3$

$\therefore\ L = 6$

Clave D

5. $L = \frac{2\operatorname{sen}\theta\cos\theta\cos2\theta\cos4\theta}{2}$

$L = \frac{2\operatorname{sen}2\theta\cos2\theta\cos4\theta}{2.2}$

$L = \frac{2\operatorname{sen}4\theta\cos4\theta}{4.2}$

$L = \frac{\operatorname{sen}8\theta}{8} = \frac{\operatorname{sen}\left(8 \cdot \frac{\pi}{24}\right)}{8} = \frac{\operatorname{sen}\left(\frac{\pi}{3}\right)}{8}$

$L = \frac{\operatorname{sen}(60°)}{8} = \frac{\frac{\sqrt{3}}{2}}{8} = \frac{\sqrt{3}}{16} = \frac{\sqrt{3}}{2^4}$

$\therefore\ L = \sqrt{3}\ .\ 2^{-4}$

Clave D

6. $C = \operatorname{sen}\phi\cos\phi\cos2\phi\cos4\phi\cos8\phi$

$C = \frac{2}{2}\operatorname{sen}\phi\cos\phi\cos2\phi\cos4\phi\cos8\phi$

$C = \frac{2}{4}\operatorname{sen}2\phi\cos2\phi\cos4\phi\cos8\phi$

$C = \frac{2}{8}\operatorname{sen}4\phi\cos4\phi\cos8\phi$

$C = \frac{2}{16}\operatorname{sen}8\phi\cos8\phi$

$C = \frac{\operatorname{sen}16\phi}{16} = \frac{\operatorname{sen}16.\frac{\pi}{32}}{16} = \frac{\operatorname{sen}\frac{\pi}{2}}{16}$

$\therefore\ C = \frac{1}{16}$

Clave E

7. $C = \frac{1 + \cos2\theta + \operatorname{sen}2\theta}{\operatorname{sen}\theta + \cos\theta}$

$\cos2\theta = \cos^2\theta - \operatorname{sen}^2\theta$

$\operatorname{sen}2\theta = 2\operatorname{sen}\theta\cos\theta$

$1 = \cos^2\theta + \operatorname{sen}^2\theta$

$C = \frac{\cos^2\theta + \operatorname{sen}^2\theta + \cos^2\theta - \operatorname{sen}^2\theta + 2\operatorname{sen}\theta\cos\theta}{\operatorname{sen}\theta + \cos\theta}$

$C = \frac{2\cos^2\theta + 2\operatorname{sen}\theta\cos\theta}{\operatorname{sen}\theta + \cos\theta} = \frac{2\cos\theta(\operatorname{sen}\theta + \cos\theta)}{\operatorname{sen}\theta + \cos\theta}$

$\Rightarrow C = 2\cos\theta$

Clave D

8. $L = \frac{1 - \cos2\theta - \operatorname{sen}2\theta}{2\operatorname{sen}\theta} + \cos\theta$

$\cos2\theta = \cos^2\theta - \operatorname{sen}^2\theta$

$\operatorname{sen}2\theta = 2\operatorname{sen}\theta\cos\theta$

$L = \frac{1 - \cos 2\theta - \operatorname{sen}2\theta + 2\operatorname{sen}\theta\cos\theta}{2\operatorname{sen}\theta}$

$L = \frac{1 - \cos 2\theta - \operatorname{sen}2\theta + \operatorname{sen}2\theta}{2\operatorname{sen}\theta}$

$L = \frac{\operatorname{sen}^2\theta + \cos^2\theta - (\cos^2\theta - \operatorname{sen}^2\theta)}{2\operatorname{sen}\theta}$

$L = \frac{2\operatorname{sen}^2\theta}{2\operatorname{sen}\theta}$

$\therefore\ L = \operatorname{sen}\theta$

Clave B

9. $L = 7\cot\frac{x}{2} - 5\tan\frac{x}{2} - 2\csc x$

$L = 7(\csc x + \cot x) - 5(\csc x - \cot x) - 2\csc x$

$L = 7\csc x + 7\cot x + 5\cot x - 5\csc x - 2\csc x$

$L = 12\cot x$

Clave E

10. $L = \csc2x + \csc4x + \underbrace{\csc8x + \cot8x}_{\cot4x}$

$L = \csc2x + \underbrace{\csc4x + \cot4x}_{\cot2x}$

$L = \csc2x + \cot2x = \cot x$

$\therefore\ L = \cot x$

Clave A

11. $L = \sec65° + \sec40° + \tan40°$

Por RT complementarias:

$\sec65° = \csc25°$

$\sec40° = \csc50°$

$\tan40° = \cot50°$

Reemplazando en L:

$L = \csc25° + \csc50° + \cot50°$

$L = \csc25° + \cot25°$

$L = \cot\left(\frac{25°}{2}\right)$

$L = \cot12°30'$

Clave C

12. $\cos\theta = \frac{1}{4};\ \theta \in \text{IC}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \left(\frac{1}{4}\right)}{2}}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{5}{8}} = \pm\sqrt{0,625}$

Como $\frac{\theta}{2} \in \text{IC} \Rightarrow \cos\frac{\theta}{2} > 0$

$\therefore \cos\frac{\theta}{2} = \sqrt{0,625}$

Clave E

13. $\cos\theta = -2/7 \ ;\ \theta \in \text{IIIC}$

$180° < \theta < 270°$

$90° < \frac{\theta}{2} < 135° \Rightarrow \frac{\theta}{2} \in \text{IIC}$

$\operatorname{sen}\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$

$\operatorname{sen}\frac{\theta}{2} = \pm\sqrt{\frac{1 - \left(-\frac{2}{7}\right)}{2}}$

$\operatorname{sen}\frac{\theta}{2} = \pm\sqrt{\frac{9}{14}}$

Como $\frac{\theta}{2} \in \text{IIC} \Rightarrow \operatorname{sen}\frac{\theta}{2} > 0$

$\therefore\ \operatorname{sen}\frac{\theta}{2} = \frac{3}{\sqrt{14}}$

Clave C

14. $\cos\beta = 0,8 = \frac{4}{5};\ 270° < \beta < 360°$

$\Rightarrow 135° < \frac{\beta}{2} < 180°$

$\cos\frac{\beta}{2} = \pm\sqrt{\frac{1 + \cos\beta}{2}}$

$\cos\frac{\beta}{2} = \pm\sqrt{\frac{1 + \frac{4}{5}}{2}}$

$\cos\frac{\beta}{2} = \pm\sqrt{\frac{9}{10}}$

Como $\frac{\beta}{2} \in \text{IIC} \Rightarrow \cos\frac{\beta}{2} < 0$

$\therefore \cos\frac{\beta}{2} = -\sqrt{0,9}$

Clave D

15. $\operatorname{sen}x = \frac{1}{3}$

Sabemos:

$\operatorname{sen}3x = 3\operatorname{sen}x - 4\operatorname{sen}^3x$

$\operatorname{sen}3x = 3\left(\frac{1}{3}\right) - 4\left(\frac{1}{3}\right)^3$

$\operatorname{sen}3x = 1 - 4\left(\frac{1}{27}\right) = 1 - \frac{4}{27}$

$\therefore\ \operatorname{sen}3x = \frac{23}{27}$

Clave C

16. $\cos x = \frac{1}{4}$

Sabemos:

$\cos3x = \cos^3x - 3\cos x$

$\cos3x = 4\left(\frac{1}{4}\right)^3 - 3\left(\frac{1}{4}\right)$

$$\cos 3x = 4\left(\frac{1}{64}\right) - \frac{3}{4}$$

$$\cos 3x = \frac{1}{16} - \frac{3}{4} = \frac{1-12}{16} = -\frac{11}{16}$$

$$\therefore \cos 3x = -\frac{11}{16}$$

Clave B

17. $\tan x = 2$

Sabemos:

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

$$\tan 3x = \frac{3(2) - (2)^3}{1 - 3(2)^2} = \frac{-2}{-11} = \frac{2}{11}$$

$$\therefore \tan 3x = \frac{2}{11}$$

Clave D

18. $E = 3\mathrm{sen}10° - 4\mathrm{sen}^3 10°$

Sabemos:

$\mathrm{sen}3x = 3\mathrm{sen}x - 4\mathrm{sen}^3 x$

Para: $x = 10°$

$\mathrm{sen}(3 \,.\, 10°) = 3\mathrm{sen}10° - 4\mathrm{sen}^3 10°$

$\Rightarrow E = \mathrm{sen}(30°) = \frac{1}{2}$

Clave C

19. $E = 3\mathrm{sen}15° - 4\mathrm{sen}^3 15°$

Sabemos:

$\mathrm{sen}3x = 3\mathrm{sen}x - 4\mathrm{sen}^3 x$

Para: $x = 15°$

$\mathrm{sen}(3 \,.\, 15°) = 3\mathrm{sen}15° - 4\mathrm{sen}^3 15°$

$\therefore E = \mathrm{sen}45° = \frac{\sqrt{2}}{2}$

Clave C

20. $3\mathrm{sen}x - 2 = 0$

$\Rightarrow \quad \mathrm{sen}x = \frac{2}{3}$

Luego:

$\mathrm{sen}3x = 3\mathrm{sen}x - 4\mathrm{sen}^3 x$

$$\mathrm{sen}3x = 3\left(\frac{2}{3}\right) - 4\left(\frac{2}{3}\right)^3$$

$$\mathrm{sen}3x = 2 - 4 \,.\, \frac{8}{27} = \frac{22}{27}$$

$$\therefore \mathrm{sen}3x = \frac{22}{27}$$

Clave E

Resolución de problemas

21. Dato: $\mathrm{sen}\theta = \frac{3}{5} \Rightarrow \cos 2\theta = 1 - 2\mathrm{sen}^2\theta$

$$\cos 2\theta = 1 - 2\left(\frac{3}{5}\right)^2$$

$$\therefore \cos 2\theta = \frac{7}{25}$$

Clave E

22. Dato: $\cos\theta = 1/3 \Rightarrow \cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 2(1/3)^2 - 1$

$\therefore \cos 2\theta = -7/9$

Clave B

Nivel 2 (página 77) Unidad 4

Comunicación matemática

23.

24.

Razonamiento y demostración

25.
$$(\mathrm{sen}\theta - \cos\theta)^2 = \left(\frac{1}{2}\right)^2$$

$$\mathrm{sen}^2\theta + \cos^2\theta - 2\mathrm{sen}\theta\cos\theta = \frac{1}{4}$$

$$1 - \mathrm{sen}2\theta = \frac{1}{4}$$

$$\mathrm{sen}2\theta = \frac{3}{4}$$

Clave B

26.
$$C = \frac{\mathrm{sen}2\varphi \cot\varphi}{2} + \mathrm{sen}^2\varphi$$

$$C = \frac{2\mathrm{sen}\varphi\cos\varphi \,.\, \frac{\cos\varphi}{\mathrm{sen}\varphi}}{2} + \mathrm{sen}^2\varphi$$

$$C = \cos\varphi \,.\, \cos\varphi + \mathrm{sen}^2\varphi$$

$$\therefore C = \cos^2\varphi + \mathrm{sen}^2\varphi = 1$$

Clave A

27.
$$L = \frac{\mathrm{sen}2\theta\tan\theta}{2} - \cos^2\theta$$

$$L = \frac{2\mathrm{sen}\theta\cos\theta \,.\, \frac{\mathrm{sen}\theta}{\cos\theta}}{2} - \cos^2\theta$$

$$L = \mathrm{sen}\theta \,.\, \mathrm{sen}\theta - \cos^2\theta = \mathrm{sen}^2\theta - \cos^2\theta$$

$$L = -(\cos^2\theta - \mathrm{sen}^2\theta)$$

$$L = -\cos 2\theta$$

Clave E

28.
$$C = \frac{\cos 2\theta - \cos^2\theta}{\cos 2\theta + \mathrm{sen}^2\theta}$$

$$C = \frac{\cos^2\theta - \mathrm{sen}^2\theta - \cos^2\theta}{\cos^2\theta - \mathrm{sen}^2\theta + \mathrm{sen}^2\theta}$$

$$C = -\frac{\mathrm{sen}^2\theta}{\cos^2\theta} = -\tan^2\theta$$

Clave C

29.
$$L = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$L = \frac{\mathrm{sen}^2\theta + \cos^2\theta - (\cos^2\theta - \mathrm{sen}^2\theta)}{\mathrm{sen}^2\theta + \cos^2\theta + \cos^2\theta - \mathrm{sen}^2\theta}$$

$$L = \frac{\mathrm{sen}^2\theta + \cos^2\theta - \cos^2\theta + \mathrm{sen}^2\theta}{2\cos^2\theta}$$

$$\therefore L = \frac{2\mathrm{sen}^2\theta}{2\cos^2\theta} = \tan^2\theta$$

Clave C

30.
$$(\mathrm{sen}\phi + \cos\phi)^2 = n^2$$

$$\mathrm{sen}^2\phi + \cos^2\phi + 2\mathrm{sen}\phi\cos\phi = n^2$$

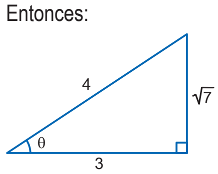
$$1 + \mathrm{sen}2\phi = n^2$$

$$\Rightarrow \mathrm{sen}2\phi = n^2 - 1$$

Clave C

31. $\tan\theta = \frac{\sqrt{7}}{3}$; $0° < \theta < 90°$

Entonces:



$\Rightarrow \cos\theta = \frac{3}{4}$

Luego:

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\left(\frac{3}{4}\right)}{1+\left(\frac{3}{4}\right)}} = \pm\sqrt{\frac{1}{7}}$$

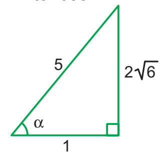
Como $\frac{\theta}{2} \in \mathrm{IC} \Rightarrow \tan\frac{\theta}{2} > 0$

$$\therefore \tan\frac{\theta}{2} = \frac{1}{\sqrt{7}}$$

Clave B

32. $\tan\alpha = 2\sqrt{6}; 0° < \alpha < 90°$

Entonces:



$\Rightarrow \cos\alpha = \frac{1}{5}$

Luego:

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\left(\frac{1}{5}\right)}{2}} = \pm\sqrt{\frac{6}{10}}$$

Como $\frac{\alpha}{2} \in \mathrm{IC} \Rightarrow \cos\frac{\alpha}{2} > 0$

$$\therefore \cos\frac{\alpha}{2} = \sqrt{0{,}6}$$

Clave E

33. $\tan\theta = \frac{\sqrt{33}}{4}$; $180° < \theta < 270°$

$$\Rightarrow 90° < \frac{\theta}{2} < 135°$$

Elevando al cuadrado:

$$\tan^2\theta = \frac{33}{16}$$

$$\sec^2\theta - 1 = \frac{33}{16}$$

$$\sec^2\theta = \frac{49}{16}$$

Como $\theta \in \mathrm{IIIC} \Rightarrow \sec\theta < 0$

$\sec\theta = -\frac{7}{4} \Rightarrow \cos\theta = -\frac{4}{7}$

Luego:

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1-\frac{4}{7}}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{3}{14}}$$

Como $\frac{\theta}{2} \in \mathrm{IIC} \Rightarrow \cos\frac{\theta}{2} < 0$

$$\therefore \cos\frac{\theta}{2} = -\sqrt{\frac{3}{14}}$$

Clave D

34. $\mathrm{sen}\beta = \frac{\sqrt{11}}{6}$

$450° < \beta < 540° \Rightarrow \beta \in \mathrm{IIC}$

$225° < \frac{\beta}{2} < 270°$

Elevando al cuadrado:

$$\text{sen}^2\beta = \frac{11}{36}$$

$$1 - \cos^2\beta = \frac{11}{36}$$

$$\cos^2\beta = \frac{25}{36}$$

Como $\beta \in \text{IIC} \Rightarrow \cos\beta < 0$

$$\text{Cos}\beta = -\frac{5}{6}$$

Luego:

$$\tan\frac{\beta}{2} = \pm\sqrt{\frac{1-\cos\beta}{1+\cos\beta}}$$

$$\tan\frac{\beta}{2} = \pm\sqrt{\frac{1-\left(-\frac{5}{6}\right)}{1+\left(-\frac{5}{6}\right)}}$$

$$\tan\frac{\beta}{2} = \pm\sqrt{11}$$

Como $\frac{\beta}{2} \in \text{IIIC} \Rightarrow \tan\frac{\beta}{2} > 0$

$$\therefore\ \tan\frac{\beta}{2} = \sqrt{11}$$

Clave A

35. $C = \dfrac{\csc 40° + \csc 80° + \csc 160°}{\cot 20°}$

$\cot 20° = \csc 40° + \cot 40°$
$\cot 40° = \csc 80° + \cot 80°$
$\cot 80° = \csc 160° + \underbrace{\cot 160°}_{-\cot 20°}$

Sumando y ordenando tenemos:
$2\cot 20° = \csc 40° + \csc 80° + \csc 160°$
Reemplazando:

$$C = \frac{2\cot 20°}{\cot 20°} \qquad \therefore\ C = 2$$

Clave C

36. $K = \left[\cot\frac{\alpha}{2} - \tan\frac{\alpha}{2}\right]\tan\alpha$

Se sabe:

$\cot\frac{\alpha}{2} = \csc\alpha + \cot\alpha$
$\tan\frac{\alpha}{2} = \csc\alpha - \cot\alpha \quad (-)$

$\Rightarrow \cot\frac{\alpha}{2} - \tan\frac{\alpha}{2} = 2\cot\alpha$

Reemplazando en la expresión:
$K = (2\cot\alpha)\tan\alpha$
$K = 2\underbrace{\tan\alpha\cot\alpha}_{1} = 2$

$\therefore K = 2$

Clave C

37. $E = \dfrac{\text{sen}^3x + \text{sen}3x}{\text{sen}x}$

Sabemos:
$\text{sen}3x = 3\text{sen}x - 4\text{sen}^3x$

$$E = \frac{\text{sen}^3x + 3\text{sen}x - 4\text{sen}^3x}{\text{sen}x}$$

$$E = \frac{3\text{sen}x - 3\text{sen}^3x}{\text{sen}x} = \frac{3\text{sen}x(1-\text{sen}^2x)}{\text{sen}x}$$

$\therefore\ E = 3\cos^2x$

Clave D

38. $E = (\cos3x - 4\cos^3x)\sec x$

Sabemos:
$\cos3x = 4\cos^3x - 3\cos x$
$E = (4\cos^3x - 3\cos x - 4\cos^3x).\ \dfrac{1}{\cos x}$
$E = (-3\cos x).\ \dfrac{1}{\cos x}$
$\therefore E = -3$

Clave D

39. $E = \dfrac{\text{sen}3x}{\text{sen}x} - \dfrac{\cos 3x}{\cos x}$

Sabemos:
$\text{sen}3x = 3\text{sen}x - 4\text{sen}^3x$
$\cos3x = 4\cos^3x - 3\cos x$

$$E = \frac{3\text{sen}x - 4\text{sen}^3x}{\text{sen}x} - \frac{(4\cos^3x - 3\cos x)}{\cos x}$$

$$E = \frac{\text{sen}x(3-4\text{sen}^2x)}{\text{sen}x} - \frac{\cos x(4\cos^2x - 3)}{\cos x}$$

$E = 3 - 4\text{sen}^2x - 4\cos^2x + 3$
$E = 6 - 4\underbrace{(\text{sen}^2x + \cos^2x)}_{1}$

$E = 6 - 4(1)$

$\therefore E = 2$

Clave B

Resolución de problemas

40. Dato: $\cos\theta = 1/2 \Rightarrow \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}}$$

$$\cot\frac{\theta}{2} = \pm\frac{\sqrt{3}}{3}$$

$$\therefore\ \cot\frac{\theta}{2} = \sqrt{3}$$

Clave A

41. Dato: $\text{sen}\theta = \frac{3}{5} \Rightarrow \cos\theta = \frac{4}{5}$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\frac{4}{5}}{1+\frac{4}{5}}}$$

$$\therefore\ \tan\frac{\theta}{2} = \frac{1}{3}$$

Clave C

Nivel 3 (página 78) Unidad 4

Comunicación matemática

42.
43.

Razonamiento y demostración

44. Dato: $a + b + c = \pi \Rightarrow b + c = \pi - a$

Piden:
$\text{sen}(3a + 2b + 2c)\text{sen}(a + 2b + 2c) + \cos(b + c)\cos(b + 2a + c)$
Reemplazando:
$\text{sen}(3a + 2(\pi - a))\text{sen}(a + 2(\pi - a)) + \cos(\pi - a)\cos(2a + \pi - a)$
$= \text{sen}(2\pi + a)\text{sen}(2\pi - a) + \cos(\pi - a)\cos(\pi + a)$
$= \text{sen}a(-\text{sen}a) + (-\cos a)(-\cos a)$
$= -\text{sen}^2a + \cos^2a$
$= \cos2a$

Clave D

45. $A + B + C = 180° \Rightarrow A + B = 180° - C$

Además:
$\text{sen}(A + B)\cos(A + B) = -\frac{1}{2}$
$\text{sen}(180° - C)\cos(180° - C) = -\frac{1}{2}$
$\text{sen}C(-\cos C) = -\frac{1}{2}$
$2\text{sen}C\cos C = 1$
$\text{sen}2C = 1$
$2C = 90°$
$\therefore C = 45°$
Piden: $1 + \tan C = 1 + \tan 45° = 1 + 1 = 2$

Clave C

46. $U = \sec A\left[\left(\cos\frac{A}{2} + \text{sen}\frac{A}{2}\right)^2 - \text{sen}A\right]$

$U = \sec A\left[\cos^2\frac{A}{2} + \text{sen}^2\frac{A}{2} + 2\text{sen}\frac{A}{2}\cos\frac{A}{2} - \text{sen}A\right]$

$U = \sec A[1 + \text{sen}A - \text{sen}A] = \sec A$

$N = \text{sen}A\left[\left(\cos\frac{A}{4} + \text{sen}\frac{A}{4}\right)^2 - \text{sen}\frac{A}{2}\right]$

$N = \text{sen}A\left[\cos^2\frac{A}{2} + \text{sen}^2\frac{A}{4} + 2\text{sen}\frac{A}{4}\cos\frac{A}{4} - \text{sen}\frac{A}{2}\right]$

$N = \text{sen}A\left[1 + \text{sen}\frac{A}{2} - \text{sen}\frac{A}{2}\right] = \text{sen}A$

$I = \cos A\left[\left(\cos\frac{A}{2k} + \text{sen}\frac{A}{2k}\right)^2 - \text{sen}\frac{A}{k}\right]$

$I = \cos A\left[\cos^2\frac{A}{2k} + \text{sen}^2\frac{A}{2k} + 2\text{sen}\frac{A}{2k}\cos\frac{A}{2k} - \text{sen}\frac{A}{k}\right]$

$I = \cos A\left[1 + \text{sen}\frac{A}{k} - \text{sen}\frac{A}{k}\right] = \cos A$

Piden:
$U - N + I - \dfrac{1}{\cos A}$
$= \sec A - \text{sen}A + \cos A - \sec A$
$= \cos A - \text{sen}A$

Clave D

47. $E = A\cos^2\frac{x}{2} + B\cos x$

$E = \frac{A}{2}\left(2\cos^2\frac{x}{2}\right) + B\cos x$

$E = \frac{A}{2}(1 + \cos x) + B\cos x$

$E = \frac{A}{2} + \left(\frac{A}{2} + B\right)\cos x$

Entonces:

$F(x) = \frac{A}{2} + \left(\frac{A}{2} + B\right)\cos x$

Se sabe:
$-1 \leq \cos x \leq 1$

Asumimos que A y B son positivos:

$-\frac{A}{2} - B \leq \left(\frac{A}{2} + B\right)\cos x \leq \frac{A}{2} + B$

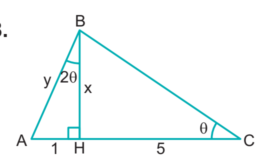
$-\frac{A}{2} - B + \frac{A}{2} \leq \frac{A}{2} + \left(\frac{A}{2} + B\right)\cos x \leq \frac{A}{2} + B + \frac{A}{2}$

$-B \leq \frac{A}{2} + \left(\frac{A}{2} + B\right)\cos x \leq A + B$

Piden: $-B + A + B = A$

Clave B

48.



$\tan\theta = \dfrac{x}{5}$

$\tan2\theta = \dfrac{1}{x}$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \Rightarrow \frac{1}{x} = \frac{\frac{2x}{5}}{1-\left(\frac{x}{5}\right)^2}$$

$$\frac{1}{x} = \frac{\frac{2x}{5}}{\frac{25-x^2}{25}} \Rightarrow \frac{1}{x} = \frac{2x\ .\ 25}{5(25-x^2)}$$

$$25 - x^2 = 10x^2$$
$$25 = 11x^2$$
$$x^2 = \frac{25}{11}$$

Del triángulo ABH:

$y^2 = x^2 + 1$

$y^2 = \frac{25}{11} + 1 = \frac{36}{11}$

$$\cos 2\theta = \frac{x}{y} = \sqrt{\frac{x^2}{y^2}} = \sqrt{\frac{\frac{25}{11}}{\frac{36}{11}}} = \sqrt{\frac{25\ .\ 11}{36\ .\ 11}}$$

$$\therefore\ \cos 2\theta = \sqrt{\frac{25}{36}} = \frac{5}{6}$$

Clave C

49. $P = \cot\alpha\tan\left(\frac{\alpha}{2}\right)(1+\cos\alpha)$

Por propiedad: $\tan\frac{\alpha}{2} = \csc\alpha - \cot\alpha$

$P = \cot\alpha(\csc\alpha - \cot\alpha)(1+\cos\alpha)$

$P = \cot\alpha\left(\frac{1}{\operatorname{sen}\alpha} - \frac{\cos\alpha}{\operatorname{sen}\alpha}\right)(1+\cos\alpha)$

$P = \cot\alpha\left(\frac{1-\cos\alpha}{\operatorname{sen}\alpha}\right)(1+\cos\alpha)$

$P = \cot\alpha\left(\frac{1-\cos^2\alpha}{\operatorname{sen}\alpha}\right) = \frac{\cos\alpha}{\operatorname{sen}\alpha}\cdot\frac{\operatorname{sen}^2\alpha}{\operatorname{sen}\alpha}$

$\therefore P = \cos\alpha$

Clave A

50. $\cos^2\alpha = \frac{4}{9};\ \alpha \in \langle 180°; 270°\rangle$

$\Rightarrow \cos\alpha = -\frac{2}{3}\quad (\alpha \in \text{IIIC})$

Luego:

$\operatorname{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$\operatorname{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\left(-\frac{2}{3}\right)}{2}} = \pm\sqrt{\frac{5}{6}}$

Como $\frac{\alpha}{2} \in \text{IIC} \Rightarrow \operatorname{sen}\frac{\alpha}{2} > 0$

$\operatorname{sen}\frac{\alpha}{2} = \sqrt{\frac{5}{6}}$

Piden:

$\sqrt{30}\operatorname{sen}\frac{\alpha}{2} = \sqrt{30}\ .\ \sqrt{\frac{5}{6}} = \sqrt{\frac{30\ .\ 5}{6}} = \sqrt{25}$

$\therefore \sqrt{30}\operatorname{sen}\frac{\alpha}{2} = 5$

Clave E

51. $M = \cot x + \cos x(\tan x - \tan\frac{x}{2})$

$M = \cot x + \cos x(\tan x - \csc x + \cot x)$

$M = \cot x + \cos x\ .\ \frac{\operatorname{sen}x}{\cos x} - \frac{\cos x}{\operatorname{sen}x} + \cos x\ .\ \frac{\cos x}{\operatorname{sen}x}$

$M = \cot x + \operatorname{sen}x - \cot x + \frac{\cos^2 x}{\operatorname{sen}x}$

$M = \frac{\operatorname{sen}^2 + \cos^2 x}{\operatorname{sen}x} = \frac{1}{\operatorname{sen}x} = \csc x$

$\therefore M = \csc x$

Clave D

52. $\sec x = \cot A\cot B$

$\Rightarrow \cos x = \tan A\tan B$

Piden:

$\tan^2\frac{x}{2} = \frac{1-\cos x}{1+\cos x} = \frac{1-\tan A\tan B}{1+\tan A\tan B}$

$\tan^2\frac{x}{2} = \frac{1-\frac{\operatorname{sen}A\operatorname{sen}B}{\cos A\cos B}}{1+\frac{\operatorname{sen}A\operatorname{sen}B}{\cos A\cos B}}$

$= \frac{\cos A\cos B - \operatorname{sen}A\operatorname{sen}B}{\cos A\cos B + \operatorname{sen}A\operatorname{sen}B}$

$\tan^2\frac{x}{2} = \frac{\cos(A+B)}{\cos(A-B)} = \cos(A+B)\ .\ \sec(A-B)$

$\therefore \tan^2\frac{x}{2} = \cos(A+B)\sec(A-B)$

Clave B

53. $\tan\frac{x}{2} + \tan\frac{x}{4} = 2\csc x$

$\tan\frac{x}{4} = \csc x + \underbrace{\csc x - \tan\frac{x}{2}}_{\cot x}$

$\tan\frac{x}{4} = \underbrace{\csc x + \cot x}$

$\tan\frac{x}{4} = \cot\frac{x}{2}$

$\Rightarrow \underbrace{\tan\frac{x}{2}}\ .\ \tan\frac{x}{4} = 1$

$\frac{2\tan\frac{x}{4}}{1-\tan^2\frac{x}{4}}\ .\ \tan\frac{x}{4} = 1$

$\underbrace{\tan^2\frac{x}{4}} = \frac{1}{3}$

$\frac{1-\cos\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{3} \Rightarrow 4\cos\frac{x}{2} = 2$

$\therefore \cos\frac{x}{2} = \frac{1}{2}$

Clave A

54. $P = \tan\frac{x}{2} + 2\operatorname{sen}^2\frac{x}{2}\cot x$

$P = \tan\frac{x}{2} + 2\left(\frac{1-\cos x}{2}\right)\cot x$

$P = \tan\frac{x}{2} + \cot x - \cos x\cot x$

$P = \csc x - \cos x\cot x$

$P = \frac{1}{\operatorname{sen}x} - \frac{\cos^2 x}{\operatorname{sen}x} = \frac{1-\cos^2 x}{\operatorname{sen}x} = \frac{\operatorname{sen}^2 x}{\operatorname{sen}x}$

$\therefore P = \operatorname{sen}x$

Clave B

55. $E = 4\operatorname{sen}5°\operatorname{sen}55°\operatorname{sen}65°$

Sabemos:

$\operatorname{sen}3x = 4\operatorname{sen}x\ .\ \operatorname{sen}(60° - x)\ .\ \operatorname{sen}(60° + x)$

Luego:

$\operatorname{sen}3(5°) = 4\operatorname{sen}5°\operatorname{sen}(60° - 5°)\operatorname{sen}(60° + 5°)$

$E = \operatorname{sen}3(5°) = \operatorname{sen}15° = \frac{\sqrt{6}-\sqrt{2}}{4}$

Clave C

56. $E = \cos 20°\ .\ \cos 40°\ .\ \cos 100°$

Sabemos:

$\cos 3x = 4\cos x\cos(60° - x)\cos(60° + x)$

$\cos 3(40°) = 4\cos 40°\cos(60° - 40°)\cos(60° + 40°)$

$\underbrace{\cos 120°}_{-1/2} = \underbrace{4\cos 40°\cos 20°\cos 100°}_{E}$

$-\frac{1}{2} = 4.E \Rightarrow E = -\frac{1}{8}$

Clave B

57. $E = 4\operatorname{sen}25°\operatorname{sen}35°\operatorname{sen}85°$

Sabemos:

$\operatorname{sen}3x = 4\operatorname{sen}x\operatorname{sen}(60° - x)\operatorname{sen}(60° + x)$

Para $x = 25°$

$\operatorname{sen}(3.25°) = 4\operatorname{sen}25°\operatorname{sen}(60° - 25°)\operatorname{sen}(60° + 25°)$

$\operatorname{sen}75° = 4\operatorname{sen}25°\operatorname{sen}35°\operatorname{sen}85°$

$\Rightarrow E = \operatorname{sen}75°\quad \therefore\ E = \frac{\sqrt{6}+\sqrt{2}}{4}$

Clave D

58. $E = \cos 10°\cos 50°\cos 110°$

Pero: $\cos 110° = -\cos 70°$

$E = \cos 10°\cos 50°(-\cos 70°)$

Sabemos:

$\cos 3x = 4\cos x\cos(60° - x)\cos(60° + x)$

$\cos(3.10°) = 4\cos 10°\cos(60° - 10°)\cos(60° + 10°)$

$\cos 30° = 4(-E)$

$\frac{\sqrt{3}}{2} = 4(-E) \Rightarrow E = -\frac{\sqrt{3}}{8}$

Clave D

59. $E = \tan\frac{2\theta}{3}\tan\left(\frac{\pi-2\theta}{3}\right)\tan\left(\frac{\pi+2\theta}{3}\right)$

Se sabe:

$\tan 3x = \tan x\tan(60° - x)\tan(60° + x)$

$\tan 3x = \tan x\tan\left(\frac{\pi}{3} - x\right)\tan\left(\frac{\pi}{3} + x\right)$

Para: $x = \frac{2\theta}{3}$

$\tan\left(3.\frac{2\theta}{3}\right) = \underbrace{\tan\frac{2\theta}{3}\tan\left(\frac{\pi}{3} - \frac{2\theta}{3}\right)\tan\left(\frac{\pi}{3} + \frac{2\theta}{3}\right)}_{E}$

$\therefore\ E = \tan 2\theta$

Clave A

60. $E = \tan\frac{\theta}{6}\tan\left(\frac{2\pi-\theta}{6}\right)\tan\left(\frac{2\pi+\theta}{6}\right)$

$E = \tan\frac{\theta}{6}\tan\left(\frac{\pi}{3} - \frac{\theta}{6}\right)\tan\left(\frac{\pi}{3} + \frac{\theta}{6}\right)$

Sabemos:

$\tan 3x = \tan x\tan(60° - x)\tan(60° + x)$

Para: $x = \frac{\theta}{6}$

$\tan 3.\frac{\theta}{6} = \tan\frac{\theta}{6}\tan\left(\frac{\pi}{3} - \frac{\theta}{6}\right)\tan\left(\frac{\pi}{3} + \frac{\theta}{6}\right)$

$\therefore\ E = \tan\left(3\ .\ \frac{\theta}{6}\right) = \tan\frac{\theta}{2}$

Clave A

Resolución de problemas

61. Dato: $\sec\theta = 2 \Rightarrow \cos\theta = 1/2$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\cos 3\theta = 4\ .\ (1/2)^3 - 3\ .\ (1/2)$

$\therefore\ \cos 3\theta = -1$

Clave B

62. Dato: $\csc\theta = 5/4 \Rightarrow \operatorname{sen}\theta = 4/5$

$\operatorname{sen}3\theta = 3\operatorname{sen}\theta - 4\operatorname{sen}3\theta$

$\operatorname{sen}3\theta = 3\ .\ 4/5 - 4\ .\ (4/5)^3$

$\therefore\ \operatorname{sen}3\theta = \frac{44}{125}$

Clave C

PRACTIQUEMOS
Nivel 1 (página 82) Unidad 4

Comunicación matemática

1.
- $\text{senA} + \boxed{\text{senB}} + \boxed{\text{senC}}$

  $= 4\cos\dfrac{A}{2} \times \cos\dfrac{B}{2} \times \cos\dfrac{C}{2}$
- $\cos A + \cos B + \cos C$

  $= 4\ \boxed{\text{sen}\dfrac{A}{2}} \times \boxed{\text{sen}\dfrac{B}{2}} \times \boxed{\text{sen}\dfrac{C}{2}} + 1$
- $\text{sen}2A + \text{sen}2B + \text{sen}2C$

  $= 4\text{senA} \times \text{senB} \times \boxed{\text{senC}}$
- $\cos 2A + \cos 2B + \cos 2C$

  $= -4\cos A\ \boxed{\cos B} \times \boxed{\cos C} - 1$

2.
- $2\text{sen}30°\cos 10°$

  $= \boxed{\text{sen}40° + \text{sen}20°}$
- $2\cos 6x\,\text{sen}2x$

  $= \boxed{\text{sen}8x - \text{sen}4x}$
- $2\cos(46°)\text{sen}(-6°)$

  $= \boxed{\text{sen}40° - \text{sen}52°}$
- $-2\text{sen}\left(\dfrac{9x}{2}\right)\text{sen}\left(\dfrac{5x}{2}\right)$

  $= \boxed{\cos 7x - \cos 2x}$
- $2\cos 40°\cos b$

  $= \boxed{\cos(40° + b) + \cos(40° - b)}$

Razonamiento y demostración

3. $G = \dfrac{\text{sen}20° + \text{sen}40° + \text{sen}60°}{\cos 10° + \cos 30° + \cos 50°}$

Aplicando las transformaciones:
$\text{sen}20° + \text{sen}60° = 2\text{sen}40°\cos 20°$
$\cos 10° + \cos 50° = 2\cos 30°\cos 20°$

Reemplazando:

$$G = \frac{2\text{sen}40°\cos 20° + \text{sen}40°}{2\cos 30°\cos 20° + \cos 30°}$$

$$G = \frac{\text{sen}40°(2\cos 20° + 1)}{\cos 30°(2\cos 20° + 1)} = \frac{\text{sen}40°}{\cos 30°}$$

$$\therefore G = \frac{\text{sen}40°}{\frac{\sqrt{3}}{2}} = \frac{2\text{sen}40°}{\sqrt{3}}$$

Clave C

4. $H = \dfrac{\text{sen}7x - \text{sen}x}{\cos x - \cos 7x}$

Aplicando la transformación:
$\text{sen}7x - \text{sen}x = 2\cos(4x)\text{sen}(3x)$
$\cos x - \cos 7x = -2\text{sen}(4x)\,\text{sen}(-3x)$
$= -2\text{sen}4x(-\text{sen}3x)$
$= 2\text{sen}4x\,\text{sen}3x$

$$H = \frac{2\cos 4x\,\text{sen}3x}{2\text{sen}4x\,\text{sen}3x} = \frac{\cos 4x}{\text{sen}4x} = \cot 4x$$

Clave D

5. $M = \dfrac{\text{sen}x + \text{sen}y}{\cos x + \cos y}$

Por transformaciones:

$$\text{sen}x + \text{sen}y = 2\text{sen}\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$M = \frac{2\text{sen}\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

$$M = \frac{\text{sen}\left(\frac{x+y}{2}\right)}{\cos\left(\frac{x+y}{2}\right)}$$

$$\therefore M = \tan\left(\frac{x+y}{2}\right)$$

Clave A

6. $R = \text{sen}3x + \text{sen}5x + \text{sen}9x + \text{sen}11x$

Aplicando las transformaciones:
$\text{sen}11x + \text{sen}3x = 2\text{sen}7x\cos 4x$
$\text{sen}9x + \text{sen}5x = 2\text{sen}7x\cos 2x$

$R = 2\text{sen}7x\cos 4x + 2\text{sen}7x\cos 2x$
$R = 2\text{sen}7x(\cos 4x + \cos 2x)$

Luego:
$\cos 4x + \cos 2x = 2\cos 3x\cos x$
$\Rightarrow\ R = 2\text{sen}7x\,(2\cos 3x\cos x)$
$\therefore\ R = 4\cos x\cos 3x\,\text{sen}7x$

Clave A

7. $Q = \text{sen}47°\cos 17° - \cos 60°\cos 26°$
$2Q = 2\text{sen}47°\cos 17° - 2\cos 60°\cos 26°$
$2Q = \text{sen}64° + \text{sen}30° - 2\left(\frac{1}{2}\right)\cos 26°$

$2Q = \text{sen}64° + \text{sen}30° - \cos 26°$
$2Q = \cos 26° + \text{sen}30° - \cos 26°$
$2Q = \text{sen}30° = \frac{1}{2}$
$\therefore Q = \frac{1}{4}$

Clave E

8. $P = \sec 41°\sec 4°(\cos 37° + \sec 45°\text{sen}30°)$

$$\frac{P}{2} = \frac{\cos 37° + \sec 45°\,\text{sen}30°}{2\cos 41°\cos 4°}$$

$$\frac{P}{2} = \frac{\cos 37° + \sec 45°\,\text{sen}30°}{\cos 45° + \cos 37°}$$

$$\frac{P}{2} = \frac{\frac{4}{5} + \sqrt{2}.\left(\frac{1}{2}\right)}{\frac{\sqrt{2}}{2} + \frac{4}{5}} = 1$$

$\therefore P = 2$

Clave C

9. $E = \frac{1}{2}\csc 10° - 2\cos 20°$

$$E = \frac{1}{2\text{sen}10°} - 2\cos 20°$$

$$E = \frac{1 - 2(2\text{sen}10°\cos 20°)}{2\text{sen}10°}$$

$$E = \frac{1 - 2(\text{sen}30° - \text{sen}10°)}{2\text{sen}10°} = \frac{1 - 1 + 2\text{sen}10°}{2\text{sen}10°}$$

$$E = \frac{2\text{sen}10°}{2\text{sen}10°} = 1$$

$\therefore E = 1$

Clave A

10. $E = 2\text{sen}3x\cos 2x - \text{sen}x$
$E = \text{sen}(3x + 2x) + \text{sen}(3x - 2x) - \text{sen}x$
$E = \text{sen}5x + \text{sen}x - \text{sen}x$
$\therefore\ E = \text{sen}5x$

Clave D

11. $E = 2\text{sen}x\cos 3x + \text{sen}2x$
$E = \text{sen}(x + 3x) + \text{sen}(x - 3x) + \text{sen}2x$
$E = \text{sen}4x + \text{sen}(-2x) + \text{sen}2x$
$E = \text{sen}4x - \text{sen}2x + \text{sen}2x$
$\therefore\ E = \text{sen}4x$

Clave D

Resolución de problemas

12. Tenemos:
$M + N + P = 180°$
$\Rightarrow\ \cos M + \cos N + \cos P$

$$= 4\text{sen}\frac{M}{2}\text{sen}\frac{N}{2}\text{sen}\frac{P}{2} + 1$$

$$\frac{1}{2} + \frac{1}{2} + \cos P$$

$$= 4 \times \sqrt{\frac{1 - \cos M}{2}} \times \sqrt{\frac{1 - \cos N}{2}} \times \sqrt{\frac{1 - \cos P}{2}} + 1$$

$$\cos P = 4 \times \sqrt{\frac{1 - \frac{1}{2}}{2}} \times \sqrt{\frac{1 - \frac{1}{2}}{2}} \times \sqrt{\frac{1 - \cos P}{2}}$$

$$\cos P = 4 \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \sqrt{\frac{1 \cos P}{2}}$$

$$\cos P = \sqrt{\frac{1 - \cos P}{2}}$$

$$\cos^2 P = \frac{1 - \cos P}{2}$$

$2\cos^2 P + \cos P - 1 = 0$
$2\cos P \qquad -1$
$\cos P \qquad\ \ 1$

$(2\cos P - 1)(\cos P + 1) = 0\ (0° < M < P < 180°)$
$2\cos P - 1 = 0$
$\therefore\ \cos P = 1/2$

Clave B

13. De la función tenemos:

$$T(x) = 2\cos(x + 60°) + 2\left[\cos x\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2}\text{sen}x\right]$$

$T(x) = 2\cos(x + 60°) + 2[\cos x\cos 60° + \text{sen}x\,\text{sen}60°]$

Por transformaciones sabemos:
$\cos(x + 60°) + \cos(x - 60°) = 2\cos x\cos 60°$
$\Rightarrow\ T(x) = 2(2\cos x\cos 60°)$
$T(x) = 4\cos x\cos 60°$
$T(x) = 4\cos x(1/2) = 2\cos x$

Sabemos:
$-1 \le \cos x \le 1$
$-2 \le 2\cos x \le 2$
$-2 \le T(x) \le 2$
$\therefore\ T(x)_{máx.} = 2$

Clave D

Nivel 2 (página 82) Unidad 4

Comunicación matemática

14. • sen5x + sen2x

$$= \boxed{2\text{sen}\frac{7x}{2}\cos\frac{3x}{2}}$$

• $\cos\theta + \cos5\theta$

$$= \boxed{2\cos3\theta\cos2\theta}$$

• $-\text{sen}\alpha + \text{sen}7\alpha$

$$= \boxed{\text{sen}7\alpha - \text{sen}\alpha = 2\text{sen}3\alpha\cos4\alpha}$$

• $\cos\frac{\pi}{8} + \cos\frac{\pi}{3}$

$$= \boxed{2\cos\frac{11\pi}{48}\cos\frac{5\pi}{48}}$$

• $\text{sen}\frac{\pi}{10} + \text{sen}\frac{\pi}{9}$

$$= \boxed{2\text{sen}\frac{19\pi}{180}\cos\frac{\pi}{180}}$$

• sen2x + cos4x = No hay identidad

$\Rightarrow$ sen2x + cos4x

$= \cos(90° - 2x) + \cos4x$

$= 2\cos\left(\frac{90° - 2x + 4x}{2}\right)\cos\left(\frac{90° - 2x - 4x}{2}\right)$

$= 2\cos\left(\frac{90° + 2x}{2}\right)\cos\left(\frac{90° - 6x}{2}\right)$

$\therefore \text{sen}2x + \cos4x = \boxed{2\cos(45° + x)\cos(45° - 3x)}$

• $\text{sen}4x + \cos2x = \cos(90° - 4x) + \cos2x$

$= 2\cos\left(\frac{90° - 4x + 2x}{2}\right)\cos\left(\frac{90° - 4x - 2x}{2}\right)$

$= 2\cos\left(\frac{90° - 2x}{2}\right)\cos\left(\frac{90° - 6x}{2}\right)$

$\therefore \text{sen}4x + \cos2x = \boxed{2\cos(45° - x)\cos(45° - 3x)}$

15. • $\text{sen}3x + \cos5x = \cos(90° - 3x) + \cos5x$

$= 2\cos\left(\frac{90° - 3x + 5x}{2}\right)\cos\left(\frac{90° - 3x - 5x}{2}\right)$

$= 2\cos(45° + x)\cos(45° - 4x)$

$\boxed{V}$

• $\cos100° + \cos140°$

$= 2\cos\left(\frac{100° + 140°}{2}\right)\cos\left(\frac{100° - 140°}{2}\right)$

$= 2\cos120°\cos(-20°)$

$= 2\cos120°\cos20°$

$\boxed{V}$

• $\cos33° - \text{sen}87°$

$= \cos33° - \cos3°$

$= -2\text{sen}\left(\frac{33° + 3°}{2}\right)\text{sen}\left(\frac{33° - 3°}{2}\right)$

$= -2\text{sen}18°\text{sen}15°$

$\boxed{V}$

• $\text{sen}5\theta + \text{sen}\theta = 2\text{sen}\left(\frac{5\theta + \theta}{2}\right)\cos\left(\frac{5\theta - \theta}{2}\right)$

$= 2\text{sen}3\theta\cos2\theta$

$\boxed{F}$

• $\cos5\theta + \cos\theta = 2\cos\left(\frac{5\theta + \theta}{2}\right)\cos\left(\frac{5\theta - \theta}{2}\right)$

$= 2\cos3\theta\cos2\theta$

$\boxed{F}$

Razonamiento y demostración

16. $H = \dfrac{\text{sen}x + \text{sen}3x}{\text{sen}2x + \text{sen}4x}$

Por transformaciones:

senx + sen3x = 2sen2xcosx

sen2x + sen4x = 2sen3xcosx

$H = \dfrac{2\text{sen}2x\cos x}{2\text{sen}3x\cos x} = \dfrac{\text{sen}2x}{\text{sen}3x}$

Clave D

17. E = senA + sen2A + sen3A

$E = \underline{\text{sen}A + \text{sen}3A} + \text{sen}2A$

E = 2sen2AcosA + sen2A

E = 2sen2AcosA + 2senAcosA

E = 2cosA(sen2A + senA)

$E = 2\cos A\left(2\text{sen}\frac{3A}{2}\cos\frac{A}{2}\right)$

$\therefore E = 4\text{sen}\frac{3A}{2}\cos\frac{A}{2}\cos A$

Clave A

18. E = senx + sen3x + sen5x + sen7x

Por transformaciones:

sen7x + senx = 2sen4xcos3x

sen5x + sen3x = 2sen4xcosx

Reemplazando:

E = 2sen4xcos3x + 2sen4xcosx

E = 2sen4x(cos3x + cosx)

Luego:

cos3x + cosx = 2cos2xcosx

$\Rightarrow$ E = 2sen4x(2cos2xcosx)

$\therefore$ E = 4cosxcos2xsen4x

Clave E

19. P(x) =

sen3xcos2x + sen3xcos4x − senxcos6x

Por transformaciones:

2sen3xcos2x = sen5x + senx

$\Rightarrow \text{sen}x\cos2x = \dfrac{\text{sen}5x}{2} + \dfrac{\text{sen}x}{2}$ ...(1)

2sen3xcos4x = sen7x + sen(−x)

$\Rightarrow \text{sen}3x\cos4x = \dfrac{\text{sen}7x}{2} - \dfrac{\text{sen}x}{2}$ ...(2)

2senxcos6x = sen7x + sen(−5x)

$\Rightarrow \text{sen}x\cos6x = \dfrac{\text{sen}7x}{2} - \dfrac{\text{sen}5x}{2}$ ...(3)

Reemplazando (1), (2) y (3) en P(x) y reduciendo:

P(x) = sen5x

Piden: $P\left(\frac{\pi}{30}\right)$

$P\left(\frac{\pi}{30}\right) = \text{sen}\left(5.\frac{\pi}{30}\right) = \text{sen}\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\therefore P\left(\frac{\pi}{30}\right) = \frac{1}{2}$

Clave B

20. E = 2sen5xcosx − sen6x

E = sen(5x + x) + sen(5x − x) − sen6x

E = sen6x + sen4x − sen6x

$\therefore$ E = sen4x

Clave B

21. $H = \dfrac{2\text{sen}3x\cos x - \text{sen}4x}{2\cos5x\cos4x - \cos9x}$

Por transformaciones:

2sen3xcosx = sen4x + sen2x

2cos5xcos4x = cos9x + cosx

Reemplazando:

$H = \dfrac{\text{sen}4x + \text{sen}2x - \text{sen}4x}{\cos9x + \cos x - \cos9x}$

$H = \dfrac{\text{sen}2x}{\cos x} = \dfrac{2\text{sen}x\cos x}{\cos x}$

$\therefore$ H = 2senx

Clave A

22. $R = \dfrac{2\text{sen}40°\cos20° - \text{sen}20°}{2\cos35°\cos10° - \cos25°}$

Por transformaciones:

2sen40°cos20° = sen60° + sen20°

2cos35°cos10° = cos45° + cos25°

Reemplazando en la expresión:

$R = \dfrac{\text{sen}60° + \text{sen}20° - \text{sen}20°}{\cos45° + \cos25° - \cos25°} = \dfrac{\text{sen}60°}{\cos45°}$

$R = \dfrac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}} = \dfrac{\sqrt{3}}{\sqrt{2}} = \dfrac{\sqrt{3}}{\sqrt{2}}.\dfrac{\sqrt{2}}{\sqrt{2}}$

$\therefore R = \dfrac{\sqrt{6}}{2}$

Clave C

23. M = 2sen7θsen5θ − 2sen3θsenθ

Entonces:

2sen7θsen5θ = cos2θ − cos12θ

2sen3θsenθ = cos2θ − cos4θ

Reemplazando y reduciendo en M:

M = cos4θ − cos12θ

M = −2sen8θsen(−4θ)

$M = \text{sen}8\theta\,\dfrac{(2\text{sen}4\theta\cos4\theta)}{\cos4\theta}$

$M = \text{sen}8\theta\,.\,\dfrac{\text{sen}8\theta}{\cos4\theta} = \dfrac{\text{sen}^2 8\theta}{\cos4\theta}$

$\therefore M = \text{sen}^2 8\theta\sec4\theta$

Clave C

24. P = (sen38° + cos68°)sec8°

P = (sen38° + sen22°)sec8°

$P = 2\text{sen}\left(\frac{38° + 22°}{2}\right)\cos\left(\frac{38° - 22°}{2}\right)\sec8°$

$P = 2\text{sen}30°\underbrace{\cos8°\sec8°}_{1}$

$P = 2\left(\frac{1}{2}\right) = 1$

$\therefore$ P = 1

Clave A

Resolución de problemas

25. Tenemos: A + B + C = 180° (A y B < 90°)

$\text{sen}2A + \text{sen}2B = \text{sen}2C + 2\dfrac{\cos A\cos B}{\text{sen}C}$

Sabemos:

sen2A + sen2B − sen2C = 4cosAcosBsenC

Reemplazamos:

$4\cos A\cos B \,\mathrm{Sen}C = \dfrac{2\cos A.\cos B}{\mathrm{sen}C}$

$2\mathrm{sen}^2C = 1$

$1 - \cos 2C = 1$

$\cos 2C = 0 \Rightarrow 2C = 90°; 270°; ...$

$C = 45°; 135°; ...$

Pero $C > 90°$ y $C < 180°$.

$\Rightarrow C = 135°$

$\therefore \tan C = \tan 135° = -1$

Clave A

26. $M = \mathrm{sen}(2x + 10°)\mathrm{sen}(20° - 2x)$

$M = \dfrac{1}{2}[\cos(4x - 10°) - \cos 30°]$

$M = \dfrac{1}{2}\left[\cos(4x - 10°) - \dfrac{\sqrt{3}}{2}\right]$

Sabemos: $-1 \le \cos(4x - 10°) \le 1$

$-1 - \dfrac{\sqrt{3}}{2} \le \cos(4x - 10°) - \dfrac{\sqrt{3}}{2} \le 1 - \dfrac{\sqrt{3}}{2}$

$-\dfrac{1}{2} - \dfrac{\sqrt{3}}{4} \le \dfrac{1}{2}\cos(4x - 10°)$

$-\dfrac{\sqrt{3}}{4} \le \dfrac{1}{2} - \dfrac{\sqrt{3}}{4}$

$$\left.\begin{array}{l} M_{máx.} = \dfrac{1}{2} - \dfrac{\sqrt{3}}{4} \\ M_{mín.} = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{4} \end{array}\right\} (+) = -\dfrac{\sqrt{3}}{2}$$

Clave E

Nivel 3 (página 83) Unidad 4

Comunicación matemática

27. • $M = \mathrm{sen}10°\mathrm{sen}50° + \mathrm{sen}130°\mathrm{sen}610° - \mathrm{sen}430°\cos 280°$

$2M = 2\mathrm{sen}10°\mathrm{sen}50° + 2\mathrm{sen}130°\mathrm{sen}610° - 2\mathrm{sen}430°\cos 280°$

$2M = \cos 40° - \cos 60° + \cos 480° - \cos 740° - \mathrm{sen}710° - \mathrm{sen}150°$

$2M = \cos 40° - \cos 60° + \cos 120° - \cos 20° + \mathrm{sen}10° - \mathrm{sen}30°$

$2M = \cos 40° - \cos 60 - \cos 60° - \cos 20° + \mathrm{sen}10° - \mathrm{sen}30°$

$2M = \cos 40° - \dfrac{1}{2} - \dfrac{1}{2} - \cos 20° + \mathrm{sen}10° - 1/2$

$2M = -\dfrac{3}{2} - (2\mathrm{sen}30°\mathrm{sen}10°) + \mathrm{sen}10°$

$2M = -\dfrac{3}{2} - \mathrm{sen}10° + \mathrm{sen}10° = -\dfrac{3}{2}$

$\therefore M = -3/4$

• $N = \dfrac{24}{25}\mathrm{sen}34° - \mathrm{sen}52°\mathrm{sen}88°$

$N = \mathrm{sen}74°\mathrm{sen}34° - \mathrm{sen}52°\mathrm{sen}88°$

$N = \dfrac{1}{2}(2\mathrm{sen}74°\mathrm{sen}34° - 2\mathrm{sen}52°\mathrm{sen}88°)$

$N = \dfrac{1}{2}(\cos 40° - \cos 108°)$

$-\dfrac{1}{2}(\cos 36° - \cos 140°)$

$N = \dfrac{\cos 40°}{2} + \dfrac{\cos 72°}{2} - \dfrac{\cos 36°}{2} - \dfrac{\cos 40°}{2}$

$N = \dfrac{\cos 72° - \cos 36°}{2}$

$N = \dfrac{\underbrace{\dfrac{\sqrt{5}-1}{4} - \dfrac{\sqrt{5}+1}{4}}}{2}$

$N = -1/4$

$\therefore M = 3N$

Clave D

28. $\mathrm{sen}2A + \mathrm{sen}2B - \mathrm{sen}2C = 0$

$2\mathrm{sen}(A + B)\cos(A - B) - 2\mathrm{sen}C\cos C = 0$

$2\mathrm{sen}(A+B)\cos(A-B) + 2\mathrm{sen}(A+B)\cos(A+B) = 0$

$2\mathrm{sen}(A + B)[\cos(A - B) + \cos(A + B)] = 0$

$2\mathrm{sen}C[2\cos A\cos B] = 0$

$4\cos A\cos B\,\mathrm{sen}C = 0$

$\cos A = 0 \Rightarrow A = 90°$

$\cos B = 0 \Rightarrow B = 90°$

$\mathrm{sen}C = 0 \Rightarrow C = 0°$ o $180°$ (imposible)

$\Rightarrow A = 90°$ o $B = 90°$

Luego el triángulo es un triángulo rectángulo.

Clave B

Razonamiento y demostración

29. $E = \dfrac{\mathrm{sen}3\theta - \mathrm{sen}\theta}{\cos\theta - \cos 3\theta}$

$E = \dfrac{2\cos 2\theta\,\mathrm{sen}\theta}{-2\mathrm{sen}2\theta\,\mathrm{sen}(-\theta)} = \dfrac{2\cos 2\theta\,\mathrm{sen}\theta}{2\mathrm{sen}2\theta\,\mathrm{sen}\theta}$

$E = \dfrac{\cos 2\theta}{\mathrm{sen}2\theta} = \cot 2\theta$

$\therefore E = \cot 2\theta$

Clave B

30. $M = \mathrm{sen}(270° + x) + \cos(90° + x)$

$M = -\cos x - \mathrm{sen}x$

$N = 2\cos(360° - x) + 4\mathrm{sen}(-360° - x)$

$N = 2\cos(360° - x) + 4\mathrm{sen}(-(360° + x))$

$N = 2(\cos x) - 4\mathrm{sen}(360° + x)$

$N = 2\cos x - 4\mathrm{sen}x$

Luego:

$M + N = -5\mathrm{sen}x + \cos x$

Por propiedad:

$-\sqrt{(-5)^2 + (1)^2} \le M + N \le \sqrt{(-5)^2 + (1)^2}$

$-\sqrt{26} \le M + N \le \sqrt{26}$

$\therefore M + N \in [-\sqrt{26}; \sqrt{26}]$

Clave C

31. $K = \dfrac{\mathrm{sen}^2\dfrac{\pi}{14} + \mathrm{sen}^2\dfrac{3\pi}{14} + \mathrm{sen}^2\dfrac{5\pi}{14}}{\cos^2\dfrac{\pi}{14} + \cos^2\dfrac{3\pi}{14} + \cos^2\dfrac{5\pi}{14}}$

Multiplicando por 2 al numerador y denominador y degradando por ángulo doble:

$K = \dfrac{3 - \left(\cos\dfrac{\pi}{7} + \cos\dfrac{3\pi}{7} + \cos\dfrac{5\pi}{7}\right)}{3 + \left(\cos\dfrac{\pi}{7} + \cos\dfrac{3\pi}{7} + \cos\dfrac{5\pi}{7}\right)}$

Sea:

$S = \cos\dfrac{\pi}{7} + \cos\dfrac{3\pi}{7} + \cos\dfrac{5\pi}{7}$

Multiplicando a S por $2\mathrm{sen}\dfrac{2\pi}{7}$ y usando la transformación de producto a suma:

$2\mathrm{sen}\dfrac{2\pi}{7} \,.\, S = \mathrm{sen}\dfrac{5\pi}{7}$

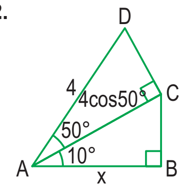
$\Rightarrow S = \dfrac{1}{2}$

Reemplazando en K:

$K = \dfrac{3 - \dfrac{1}{2}}{3 + \dfrac{1}{2}} = \dfrac{\dfrac{5}{2}}{\dfrac{7}{2}} = \dfrac{5}{7}$

Clave D

32.



Del gráfico:

$x = (4\cos 50°)\cos 10°$

$x = 2(2\cos 50°\cos 10°)$

$x = 2(\cos 60° + \cos 40°)$

$x = 2(0,5 + 0,766) = 2(1,266)$

$\therefore x = 2,532$

Clave A

33. $f(x) = \cos\left(\dfrac{2\pi}{9} + x\right)\cos\left(\dfrac{\pi}{9} - x\right)$

$2f(x) = 2\cos\left(\dfrac{2\pi}{9} + x\right)\cos\left(\dfrac{\pi}{9} - x\right)$

$2f(x) = \cos\dfrac{3\pi}{9} + \cos\left(\dfrac{\pi}{9} + 2x\right)$

$2f(x) = \cos\dfrac{\pi}{3} + \cos\left(\dfrac{\pi}{9} + 2x\right)$

$2f(x) = \dfrac{1}{2} + \cos\left(\dfrac{\pi}{9} + 2x\right)$

Sabemos:

$-1 \le \cos\left(\dfrac{\pi}{9} + 2x\right) \le 1$

$-\dfrac{1}{2} \le \dfrac{1}{2} + \cos\left(\dfrac{\pi}{9} + 2x\right) \le \dfrac{3}{2}$

$-\dfrac{1}{2} \le 2f(x) \le \dfrac{3}{2}$

$-\dfrac{1}{4} \le f(x) \le \dfrac{3}{4}$

$\therefore f(x)_{máx.} = \dfrac{3}{4}$

Clave D

34. $K = \cos\dfrac{9\pi}{14} + \cos\dfrac{3\pi}{14} + \cos\dfrac{\pi}{14}$

$K = \cos\dfrac{\pi}{14} + \cos\dfrac{3\pi}{14} - \cos\dfrac{5\pi}{14}$

Elevando al cuadrado:

$K^2 = S_1 + S_2$

Donde:

$S_1 = \cos^2\dfrac{\pi}{14} + \cos^2\dfrac{3\pi}{14} + \cos^2\dfrac{5\pi}{14}$

Del ejercicio 31:

$2S_1 = 3 + \dfrac{1}{2}$

$S_1 = \dfrac{7}{4}$

$S_2 = 2\cos\dfrac{\pi}{14}\cos\dfrac{3\pi}{14} - 2\cos\dfrac{\pi}{14}\cos\dfrac{5\pi}{14}$

$- 2\cos\dfrac{3\pi}{14}\cos\dfrac{5\pi}{14}$

Empleando la transformación del producto a suma y reduciendo tenemos:

$S_2 = -\left(\cos\frac{6\pi}{14} + \cos\frac{8\pi}{14}\right)$

$S_2 = -\left(\cos\frac{6\pi}{14} - \cos\frac{6\pi}{14}\right) = 0$

$\Rightarrow S_2 = 0$

Reemplazando en K^2:

$K^2 = \frac{7}{4} + 0$

$\therefore K = \frac{\sqrt{7}}{2}$

Clave B

35. $E = \cos(A + B)\cos(A - B) + \text{sen}^2A$

Por propiedad:

$\cos(A + B)\cos(A - B) = \cos^2A - \text{sen}^2B$

Reemplazando en E:

$E = \cos^2A - \text{sen}^2B + \text{sen}^2A$

$E = \underbrace{\text{sen}^2A + \cos^2A}_{1} - \text{sen}^2B = 1 - \text{sen}^2B$

$\therefore E = \cos^2B$

Clave D

36. $P = \frac{\cos 5\theta - \cos\theta}{\text{sen}\theta - \text{sen}5\theta}$

$P = \frac{-2\text{sen}3\theta\text{sen}2\theta}{2\cos 3\theta\,\text{sen}(-2\theta)}$

$P = \frac{-2\text{sen}3\theta\text{sen}2\theta}{-2\cos 3\theta\text{sen}2\theta} = \frac{\text{sen}3\theta}{\cos 3\theta}$

$\therefore P = \tan 3\theta$

Clave E

Resolución de problemas

37. $P = \cos(x + y - z) + \cos(y + z - x)$

$P = 2\cos y\cos(x - z)$

$Q = \cos(x + y + z) + \cos(z + x - y)$

$Q = 2\cos(x + z)\cos y$

$P + Q = 2\cos y(\cos(x + z) + \cos(x - z))$

$P + Q = 2\cos y(2\cos x\cos z)$

$P + Q = 4\cos x\cos y\cos z$

Reemplazamos en E:

$E = \sqrt{4\cos x\cos y\cos z\,.\,\sec x\sec y\sec z}$

$E = \sqrt{4} \Rightarrow E = 2$

Clave B

38. $\frac{\alpha}{3} + \frac{\beta}{3} = \theta \Rightarrow \alpha + \beta = 3\theta$

$\beta = 3\theta - \alpha$

Sabemos además:

$\text{sen}\beta = 2\cos\alpha\text{sen}\theta$

$\text{sen}(3\theta - \alpha) = \text{sen}(\theta + \alpha) + \text{sen}(\theta - \alpha)$

$\text{sen}(3\theta - \alpha) - \text{sen}(\theta + \alpha) = \text{sen}(\theta - \alpha)$

Transformamos a producto:

$2\cos 2\theta\text{sen}(\theta - \alpha) = \text{sen}(\theta - \alpha)$

$\text{sen}(\theta - \alpha)(2\cos 2\theta - 1) = 0$

$\text{sen}(\theta + \alpha) = 0 \ \vee\ 2\cos 2\theta - 1 = 0$

$\theta - \alpha = k\pi \ \vee\ \cos 2\theta = 1/2$

(no admite por condición)

$\therefore\ \cos 2\theta = 1/2$

Clave A

39. De la condición:

$\text{sen}A + \text{sen}C = 2\text{sen}B$

$2\text{sen}\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) = 2\text{sen}B$

Como:

$A + B + C = \pi \ \Rightarrow\ \frac{A+C}{2} = \frac{\pi}{2} - \frac{B}{2}$

$\Rightarrow 2\text{sen}\left(\frac{\pi}{2} - \frac{B}{2}\right)\cos\left(\frac{A-C}{2}\right) = 2\text{sen}B$

$2\cos\frac{B}{2}\cos\left(\frac{A-C}{2}\right) = 2\left(2\text{sen}\frac{B}{2}\cos\frac{B}{2}\right)$

$\Rightarrow \cos\left(\frac{A-C}{2}\right) = 2\text{sen}\left(\frac{\pi}{2} - \frac{A+C}{2}\right)$

$\cos\left(\frac{A-C}{2}\right) = 2\cos\left(\frac{A+C}{2}\right)$

$\cos\left(\frac{A}{2} - \frac{C}{2}\right) = 2\cos\left(\frac{A}{2} + \frac{C}{2}\right)$

$\cos\frac{A}{2}\cos\frac{C}{2} + \text{sen}\frac{A}{2}\text{sen}\frac{C}{2} = 2\cos\frac{A}{2}\cos\frac{C}{2} - 2\text{sen}\frac{A}{2}\text{sen}\frac{C}{2}$

$3\text{sen}\frac{A}{2}\text{sen}\frac{C}{2} = \cos\frac{A}{2}\cos\frac{C}{2}$

$\Rightarrow \frac{\cos\frac{A}{2}\cos\frac{C}{2}}{\text{sen}\frac{A}{2}\text{sen}\frac{C}{2}} = 3 \Rightarrow \cot\frac{A}{2}\cot\frac{C}{2} = 3$

$\therefore E = 3$

Clave B

RESOLUCIÓN DE TRIÁNGULOS OBLICUÁNGULOS

PRACTIQUEMOS

Nivel 1 (página 87) Unidad 4

Comunicación matemática

1.
- $c^2 = a^2 + b^2 - 2bc\cos A$ — F
- $\cos B = \frac{a^2 - b^2 - c^2}{2ac}$ — V
- $\frac{A}{\text{sen}B} = \frac{C}{\text{sen}C} = 22$ — V
- $a^2 = b^2 + c^2 - 2abc\cos A$ — F
- $(a - c)\tan\left(\frac{A+C}{2}\right) = (a + c)\tan\left(\frac{A-C}{2}\right)$ — V

2. I. Verdadera

II. Verdadera

III. Falsa

Clave D

3. I. $a^2 + b^2 - 2ab\cos\phi$

II. $x = b\cos\alpha + c\cos\beta$

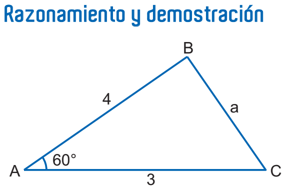
III. $\frac{x}{\text{sen}B} = \frac{a}{\text{sen}\alpha}$

IV. $x^2 = b^2 + c^2 - 2bc\cos\alpha$

V. $x = a\cos\theta + c\cos\beta$

Razonamiento y demostración

4.



Por la ley de cosenos:

$a^2 = 4^2 + 3^2 - 2(4)(3)\cos 60°$

$a^2 = 16 + 9 - 24\left(\frac{1}{2}\right)$

$a^2 = 13$

$\therefore a = \sqrt{13}$

Clave A

5. Por la ley de senos:

$\frac{BC}{\text{sen}53°} = \frac{3}{\text{sen}30°}$

$BC = \frac{3}{\text{sen}30°}\,.\,\text{sen}53° = \frac{3}{\left(\frac{1}{2}\right)}\left(\frac{4}{5}\right)$

$\therefore BC = \frac{24}{5}$

Clave B

6. Por ley de cosenos:

$(7)^2 = (8)^2 + (10)^2 - 2(8)(10)\cos A$

$49 = 64 + 100 - 160\cos A$

$\cos A = \frac{115}{160} = \frac{23}{32}$

Clave D

7. Por ley de senos:

$\frac{6}{\text{sen}74°} = \frac{15}{\text{sen}B}$

$\text{sen}B = \frac{15}{6}\,.\,\text{sen}74° = \frac{15}{6}\cdot\frac{24}{25}$

$\text{sen}B = \frac{12}{5}$

Clave D

8. Utilizamos la ley de proyecciones:

$b = 15\cos 37° + 14\cos 60°$

$b = 15 \cdot \dfrac{4}{5} + 14 \cdot \dfrac{1}{2}$

$b = 12 + 7$

$b = 19$

Clave C

9. De la ley de cosenos tenemos:

$c^2 = a^2 + b^2 - 2abcosC$

$c^2 = (3)^2 + (\sqrt{2})^2 - 2(3)(\sqrt{2}) \cdot \dfrac{\sqrt{2}}{2}$

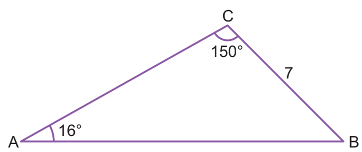
$c^2 = 9 + 2 - 6$

$c^2 = 5$

$c = \sqrt{5}$

Clave D

10.



De la ley de senos:

$\dfrac{AB}{sen150°} = \dfrac{7}{sen16°}$

$AB = \dfrac{7}{sen16°} \cdot sen150° = \dfrac{7}{\left(\dfrac{7}{25}\right)} \cdot \left(\dfrac{1}{2}\right) = 12{,}5$

$\therefore AB = 12{,}5$

Clave C

Resolución de problemas

11. $E = \dfrac{senB + senC}{senC + senA} + \dfrac{a - b}{c + a}$

De la ley de senos:

$\dfrac{a}{2R} = senA; \quad \dfrac{b}{2R} = senB; \quad \dfrac{c}{2R} = senC$

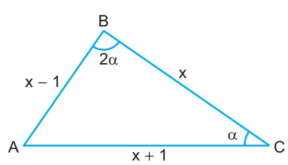
$E = \dfrac{\dfrac{b}{2R} + \dfrac{c}{2R}}{\dfrac{c}{2R} + \dfrac{a}{2R}} + \dfrac{a - b}{c + a}$

$E = \dfrac{b + c}{c + a} + \dfrac{a - b}{c + a} = \dfrac{a + c}{a + c} = 1$

$\therefore E = 1$

Clave B

12.



Por la ley de senos:

$\dfrac{x + 1}{sen2\alpha} = \dfrac{x - 1}{sen\alpha} \Rightarrow sen2\alpha = \left(\dfrac{x + 1}{x - 1}\right)sen\alpha$

$2sen\alpha cos\alpha = \left(\dfrac{x + 1}{x - 1}\right)sen\alpha$

$\Rightarrow cos\alpha = \dfrac{x + 1}{2(x - 1)}$

Por la ley de cosenos:

$(x - 1)^2 = x^2 + (x + 1)^2 - 2(x)(x + 1)cos\alpha$

$0 = x^2 + 4x - \dfrac{x(x + 1)^2}{(x - 1)}$

$\dfrac{(x + 1)^2}{(x - 1)} = x + 4$

$2x + 1 = 3x - 4$

$x = 5$

$\Rightarrow$ La longitud del lado mayor es 6.

Clave B

13. Por dado: $m\angle C = 60° \Rightarrow m\angle A + m\angle B = 120°$

Por la ley de tangente tenemos:

$\dfrac{a + b}{a - b} = \dfrac{\tan\left(\dfrac{A + B}{2}\right)}{\tan\left(\dfrac{A - B}{2}\right)}$

$\dfrac{3b + b}{3b - b} = \dfrac{\tan 60°}{\tan\left(\dfrac{A - B}{2}\right)}$

$\tan\left(\dfrac{A - B}{2}\right) = \dfrac{\sqrt{3}}{2}$

Sabemos que:

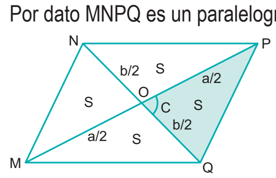
$\tan(A - B) = \dfrac{2 \cdot \tan\left(\dfrac{A - B}{2}\right)}{1 - \tan^2\left(\dfrac{A - B}{2}\right)}$

$\tan(A - B) = \dfrac{2\left(\dfrac{\sqrt{3}}{2}\right)}{1 - \left(\dfrac{\sqrt{3}}{2}\right)^2}$

$\tan(A - B) = 4\sqrt{3}$

Clave C

14. Por dato MNPQ es un paralelogramo.



Sabemos que en un paralelogramo las diagonales se cortan en su punto medio, además determinan cuatro regiones equivalentes.

En el ΔPOQ:

$S = \dfrac{OP \cdot OQ}{2} \cdot senC = \dfrac{\left(\dfrac{a}{2}\right) \cdot \left(\dfrac{b}{2}\right)}{2} \cdot senC$

$\Rightarrow S = \dfrac{ab}{8} \cdot senC \qquad ...(I)$

Piden: el área del paralelogramo.

$A_{\square} = 4S$

De (I):

$A_{\square} = 4\left(\dfrac{ab}{8} \cdot senC\right)$

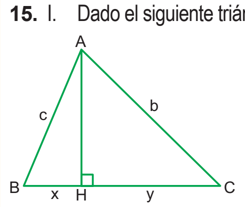
$\therefore A_{\square} = \dfrac{1}{2}absenC$

Clave D

Nivel 2 (página 88) Unidad 4

Comunicación matemática

15. I. Dado el siguiente triángulo donde AH es altura.



$\Delta AHB: x = ccosB$

$\Delta AHC: y = bcosC$

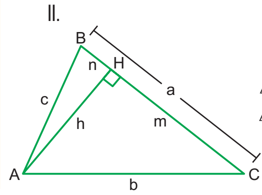
$\Rightarrow a = x + y$

$= ccosB + bcosC$

$a = ccosB + bcosC$

(V)

II.



$\Delta AHC: b^2 = h^2 + m^2 \;...(1)$

$\Delta AHB:$

$n = ccosB \quad ...(2)$

$h = csenB$

Además:

$a = m + n \Rightarrow m = a - ccosB \;...(3)$

Reemplazamos (2) y (3) en (1):

$b^2 = c^2sen^2B + (a - ccosB)^2$

$b^2 = c^2sen^2B + a^2 - 2accosB + c^2cos^2B$

$b^2 = a^2 + c^2 - 2accosB$ (F)

III. De la ley de senos:

$\dfrac{a}{senA} = \dfrac{c}{senC} \Rightarrow \dfrac{a}{c} = \dfrac{senA}{senC}$

$\Rightarrow \dfrac{a + c}{a - c} = \dfrac{senA + senC}{senA - senC}$

Luego aplicamos transformaciones trigonométricas:

$\dfrac{a + c}{a - c} = \dfrac{2sen\left(\dfrac{A + C}{2}\right)cos\left(\dfrac{A - C}{2}\right)}{2\cos\left(\dfrac{A + C}{2}\right)sen\left(\dfrac{A - C}{2}\right)}$

$= \dfrac{\tan\left(\dfrac{A + C}{2}\right)}{\tan\left(\dfrac{A - C}{2}\right)}$

(V)

Clave B

Razonamiento y demostración

16. Por la ley de senos:

$\dfrac{AC}{sen135°} = \dfrac{\sqrt{2}}{sen37°}$

$AC = \dfrac{\sqrt{2}}{sen37°} \cdot sen135° = \dfrac{\sqrt{2}}{\left(\dfrac{3}{5}\right)}\left(\dfrac{\sqrt{2}}{2}\right)$

$\therefore AC = \dfrac{5}{3}$

Clave D

17. $K = \dfrac{a}{senA} - \dfrac{b}{senB}$

De la ley de senos:

$a = 2RsenA \wedge b = 2RsenB$

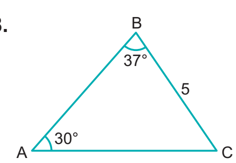
Reemplazando en la expresión:

$K = \dfrac{2RsenA}{senA} - \dfrac{2RsenB}{senB} = 2R - 2R = 0$

$\therefore K = 0$

Clave A

18.



De la ley de senos:

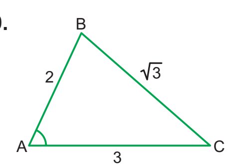
$\dfrac{AC}{sen37°} = \dfrac{5}{sen30°}$

$AC = \dfrac{5}{sen30°} \cdot sen37° = \dfrac{5}{\left(\dfrac{1}{2}\right)} \cdot \left(\dfrac{3}{5}\right) = 6$

$\therefore AC = 6$

Clave D

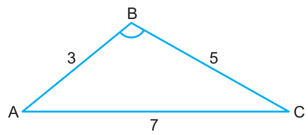
19.



Por la ley de cosenos:
$(\sqrt{3})^2 = 2^2 + 3^2 - 2(2)(3)\cos A$
$3 = 4 + 9 - 12\cos A$
$12\cos A = 10$
$\therefore \cos A = \frac{5}{6}$

Clave C

20.

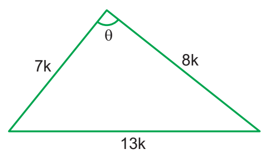


Por la ley de cosenos:
$7^2 = 3^2 + 5^2 - 2(3)(5)\cos B$
$49 = 9 + 25 - 30\cos B$
$30\cos B = -15$
$\therefore \cos B = -\frac{1}{2}$

Clave B

Resolución de problemas

21. Por dato:



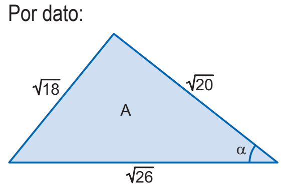
Del gráfico; θ será el mayor ángulo del triángulo ya que se le opone el mayor lado.

Por ley de cosenos:
$(13k)^2 = (7k)^2 + (8k)^2 - 2(7k)(8k)\cos\theta$

Resolviendo:
$112k^2\cos\theta = -56k^2$
$\cos\theta = -\frac{1}{2} \Rightarrow \cos\theta = \cos 120°$
$\therefore \theta = 120°$

Clave C

22. Por dato:



Por la ley de cosenos:
$(\sqrt{18})^2 = (\sqrt{20})^2 + (\sqrt{26})^2 - 2(\sqrt{20})(\sqrt{26})\cos\alpha$

$\Rightarrow 4\sqrt{130}\cos\alpha = 28 \Rightarrow \cos\alpha = \frac{7}{\sqrt{130}}$

Como: $\cos\alpha > 0 \Rightarrow 0° < \alpha < 90°$

Luego: $\cos^2\alpha = \frac{49}{130}$

$\Rightarrow 1 - \operatorname{sen}^2\alpha = \frac{49}{130}$

$\operatorname{sen}^2\alpha = \frac{81}{130}$

$\Rightarrow \operatorname{sen}\alpha = \frac{9}{\sqrt{130}}$

Piden el área de la región triangular (A):

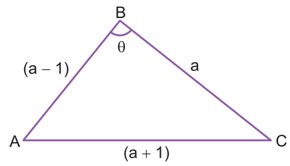
$A = \frac{(\sqrt{20}).(\sqrt{26})}{2} . \operatorname{sen}\alpha$

$\Rightarrow A = \frac{2\sqrt{130}}{2}\left(\frac{9}{\sqrt{130}}\right) = 9$

$\therefore A = 9$

Clave C

23. Por dato:



Además: $\cos\theta = \frac{1}{5}$

En el $\triangle ABC$ por ley de cosenos:
$(a+1)^2 = (a-1)^2 + a^2 - 2(a-1)(a)\cos\theta$

Luego:
$(a+1)^2 - (a-1)^2 = a^2 - 2(a-1)(a)\left(\frac{1}{5}\right)$

$4a = a^2 - 2(a-1)(a)\left(\frac{1}{5}\right)$

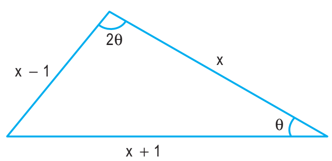
$4 = a - \frac{2(a-1)}{5}$

$20 = 5a - 2a + 2$
$18 = 3a \Rightarrow a = 6$

Piden el perímetro del $\triangle ABC$:
$2p = (a-1) + a + (a+1) = 3a$
$\Rightarrow 2p = 3(6) = 18$
$\therefore 2p = 18$

Clave D

24. Graficamos de acuerdo a los datos:



Nos piden: $\frac{x+1}{x-1}$

Aplicamos Ley de senos:

$\frac{x+1}{\operatorname{sen}2\theta} = \frac{x-1}{\operatorname{sen}\theta} \Rightarrow \frac{x+1}{x-1} = \frac{\operatorname{sen}2\theta}{\operatorname{sen}\theta}$

$\Rightarrow \frac{x+1}{x-1} = \frac{2\operatorname{sen}\theta\cos\theta}{\operatorname{sen}\theta}$

$\Rightarrow \frac{x+1}{x-1} = 2\cos\theta$

Clave C

Nivel 3 (página 89) Unidad 4

Comunicación matemática

25.
I. F
II. V
III. F
IV. V

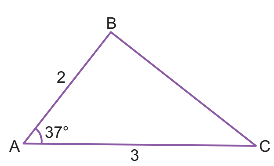
Razonamiento y demostración

26. $E = b\cos C + c\cos B + a\cos B + b\cos A - a$

Por la ley de proyecciones:
$a = b\cos C + c\cos B$
$c = a\cos B + b\cos A$
Reemplazando en la expresión:
$E = (a) + (c) - a = c$
$\therefore E = c$

Clave C

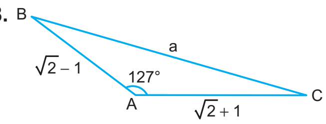
27.



Por la ley de cosenos:
$(BC)^2 = 2^2 + 3^2 - 2(2)(3)\cos 37°$
$(BC)^2 = 4 + 9 - 12\left(\frac{4}{5}\right)$
$(BC)^2 = 13 - \frac{48}{5} = \frac{17}{5}$
$\therefore BC = \sqrt{\frac{17}{5}}$

Clave D

28.



Por la ley de cosenos:
$a^2 = (\sqrt{2}-1)^2 + (\sqrt{2}+1)^2$
$-2(\sqrt{2}-1)(\sqrt{2}+1)\cos 127°$

$a^2 = 3 - 2\sqrt{2} + 3 + 2\sqrt{2} - 2(2-1)\left(-\frac{3}{5}\right)$

$a^2 = 6 + \frac{6}{5} = \frac{36}{5}$

$a = \frac{6}{\sqrt{5}} . \frac{\sqrt{5}}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$

$\therefore a = \frac{6\sqrt{5}}{5}$

Clave E

29. $E = ab\cos C + bc\cos A + ac\cos B$

De la ley de cosenos:
$a^2 = b^2 + c^2 - 2bc\cos A$

$\Rightarrow bc\cos A = \frac{b^2 + c^2 - a^2}{2}$

De la misma forma:

$ac\cos B = \frac{a^2 + c^2 - b^2}{2}$

$ab\cos C = \frac{a^2 + b^2 - c^2}{2}$

Reemplazando en la expresión:

$E = \frac{a^2 + b^2 - c^2}{2} + \frac{b^2 + c^2 - a^2}{2} + \frac{a^2 + c^2 - b^2}{2}$

$E = \frac{a^2 + b^2 + c^2}{2}$

Por dato: $a^2 + b^2 + c^2 = m$

$\therefore E = \frac{m}{2}$

Clave D

30. Por dato:
$a + b + c = 24 \quad \wedge \quad R = 5$

Piden:
$N = \operatorname{sen}A + \operatorname{sen}B + \operatorname{sen}C$

Empleando ley de senos:

$N = \left(\frac{a}{2R}\right) + \left(\frac{b}{2R}\right) + \left(\frac{c}{2R}\right)$

$N = \frac{a + b + c}{2R}$

$\Rightarrow N = \frac{(24)}{2(5)} = \frac{12}{5} \Rightarrow N = 2,4$

Clave B

31. Por dato: $a^2 + b^2 + c^2 = 10$

Piden:
$N = bc \cdot \cos A + ac \cdot \cos B + ab \cdot \cos C$

Por ley de cosenos:
- $a^2 = b^2 + c^2 - 2bc\cos A$
  $\Rightarrow 2bc\cos A = b^2 + c^2 - a^2 \quad \ldots (I)$
- $b^2 = a^2 + c^2 - 2ac\cos B$
  $\Rightarrow 2ac\cos B = a^2 + c^2 - b^2 \quad \ldots (II)$
- $c^2 = a^2 + b^2 - 2ab\cos C$
  $\Rightarrow 2ab\cos C = a^2 + b^2 - c^2 \quad \ldots (III)$

Sumando (I), (II) y (III):

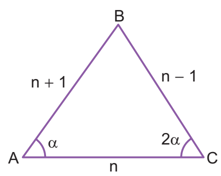
$$2\underbrace{(bc\cos A + ac\cos B + ab\cos C)}_{N} = \underbrace{a^2 + b^2 + c^2}_{10}$$

$\Rightarrow 2N = 10$
$\therefore N = 5$

Clave C

Resolución de problemas

32.



Ley de senos:

$$\frac{n+1}{\text{sen}2\alpha} = \frac{n-1}{\text{sen}\alpha}$$

$$\frac{\text{sen}2\alpha}{\text{sen}\alpha} = \frac{n+1}{n-1}$$

Luego:

$$\frac{2\text{sen}\alpha\cos\alpha}{\text{sen}\alpha} = \frac{n+1}{n-1} \Rightarrow \cos\alpha = \frac{n+1}{2(n-1)} \ldots (1)$$

Aplicando la ley de cosenos:
$(n-1)^2 = (n+1)^2 + n^2 - 2(n+1)(n)\cos\alpha \ldots (2)$

Reemplazamos (1) en (2) tenemos:

$$(n-1)^2 = (n+1)^2 + n^2 - 2(n+1)(n)\frac{(n+1)}{2(n-1)}$$

$$n^2 - 2n + 1 = n^2 + 2n + 1 + n^2 - \frac{n(n+1)^2}{(n-1)}$$

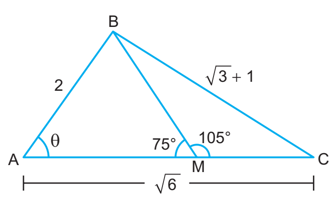
$$\frac{n(n+1)^2}{(n-1)} = n(4+n)$$

$(n+1)^2 = (n-1)(4+n)$
$n^2 + 2n + 1 = 4n + n^2 - 4 - n$
$n^2 + 2n + 1 = n^2 + 3n - 4$
$n = 5$

Luego, las longitudes de los lados son: 4; 5 y 6

Clave A

33.



Aplicamos la ley de cosenos:

$$(\sqrt{3}+1)^2 = 2^2 + (\sqrt{6})^2 - 2(2)(\sqrt{6})\cos\theta$$

$$3 + 2\sqrt{3} + 1 = 4 + 6 - 4\sqrt{6}\cos\theta$$

$$4\sqrt{6}\cos\theta = 6 - 2\sqrt{3}$$

$$\cos\theta = \frac{6-2\sqrt{3}}{4\sqrt{6}} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$\Rightarrow \theta = 75°$

Por lo tanto el triángulo ABM es isósceles.
$\Rightarrow BM = 2$

Clave B

MARATÓN MATEMÁTICA (página 90)

1. Tenemos:

$$M = \frac{\text{sen}3\theta}{\text{sen}\theta} - 2\cos2\theta$$

$$M = \frac{3\text{sen}\theta - 4\text{sen}^3\theta}{\text{sen}\theta} - 2(1 - 2\text{sen}^2\theta)$$

$M = 3 - 4\text{sen}^2\theta - 2 + 4\text{sen}^2\theta$
$\therefore M = 1$

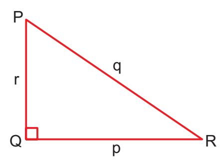
Clave A

2. Simplificamos:

$k = 4(\text{sen}^4\theta + \cos^4\theta) - \cos4\theta$
$k = 4[1 - 1/2(2\text{sen}\theta\cos\theta)^2] - (1 - 2\text{sen}^2 2\theta)$
$k = 4[1 - 1/2(\text{sen}^2 2\theta)] - 1 + 2\text{sen}^2 2\theta$
$k = 4 - 2\text{sen}^2 2\theta - 1 + 2\text{sen}^2 2\theta = 3$
$\therefore k = 3$

Clave D

3.



$$\frac{9}{q^2} = \frac{1}{p^2} + \frac{1}{r^2}$$

$$\frac{9}{q^2} = \frac{r^2 + p^2}{p^2 \cdot r^2}$$

$9p^2 \cdot r^2 = q^2 \cdot q^2$

$$\left(\frac{p}{q}\right)^2\left(\frac{r}{q}\right)^2 = \frac{1}{9}$$

$$\text{senPsenR} = \sqrt{\frac{1}{9}}$$

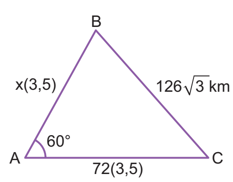
$\therefore$ senPsenR = 1/3

Clave C

$\Rightarrow 3{,}5x - 126 = 0$
$3{,}5x = 126$
$x = 36$ km/h

Clave B

4.



Luego tenemos por ley cosenos:

$$(126\sqrt{3})^2 = (72(3{,}5))^2 + (3{,}5x)^2$$
$$-2[72(3{,}5)]\,(3{,}5x)\cos60°$$

$3 \times (126)^2 = (252)^2 + (3{,}5x)^2 - 2(252)(3{,}5x)(1/2)$
$(126)^2(3 - 4) = (3{,}5x)^2 - 2(126)(3{,}5x)$
$0 = (3{,}5x)^2 - 2(126)(3{,}5x) + (126)^2$
$0 = (3{,}5x - 126)^2$

$\Rightarrow 3{,}5x - 126 = 0$
$3{,}5x = 126$
$x = 36$ km/h

Clave B

5. Empleamos las fórmulas de transformación:

$$F = \frac{\cancel{2}\cos\left(\frac{x+y}{2}\right) \times \cancel{\cos\left(\frac{x-y}{2}\right)}}{\cancel{2}\text{sen}\left(\frac{x+y}{2}\right) \times \cancel{\cos\left(\frac{x-y}{2}\right)}}$$

$$F = \frac{\cos\left(\frac{x+y}{2}\right)}{\text{sen}\left(\frac{x+y}{2}\right)} = \cot\left(\frac{x+y}{2}\right)$$

Como $x + y = 53°$

$$\Rightarrow \cot\left(\frac{x+y}{2}\right) = \cot\left(\frac{53°}{2}\right) = 2$$

$\therefore F = 2$

Clave E

6.

$$A = \frac{\csc 220° \times \text{sen}130°}{\csc 410° \times \cos 310°}$$

$$A = \frac{\csc(180° + 40°) \times \text{sen}(90° + 40°)}{\csc(450° - 40°) \times \cos(270° + 40°)}$$

$$A = \frac{-\csc 40° \times \cos 40°}{\sec 40° \times \text{sen}40°} = \frac{-\cos^2 40°}{\text{sen}^2 40°}$$

$\therefore A = -\cot^2 40°$
$A = -m^2$

Clave A

7.

$$M = \frac{2\cos2\beta\cos\beta + P\cos2\beta}{2\text{sen}2\beta\cos\beta + P\text{sen}2\beta}$$

$$M = \frac{\cos2\beta\,(2\cos\beta + P)}{\text{sen}2\beta\,(2\cos\beta + P)} = \cot2\beta$$

$\therefore M = \cot2\beta$

Clave C

8.

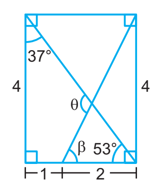
$$P = \frac{2\cos\left(\frac{x+y}{2}\right)\text{sen}\left(\frac{x-y}{2}\right)}{-2\text{sen}\left(\frac{x+y}{2}\right)\text{sen}\left(\frac{y-x}{2}\right)}$$

$$P = -\cot\left(\frac{x+y}{2}\right) = \cot\left(\frac{\pi}{6}\right) = \cot30°$$

$\therefore P = \sqrt{3}$

Clave E

9.



$\tan\theta = \tan(\beta + 53°)$

$$\tan\theta = \frac{\tan\beta + \tan 53°}{1 - \tan\beta\tan 53°} = \frac{\frac{4}{2} + \frac{4}{3}}{1 - \frac{4}{2} \times \frac{4}{3}} = \frac{\frac{10}{3}}{\frac{-5}{3}}$$

$\tan\theta = -2$
$\therefore \cot\theta = -1/2$

Clave E